Michelson-Morley, Occam and Fisher: The Radical Implications of Stable Inflation at Near-Zero Interest Rates

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Abstract

The long period of quiet inflation at near-zero interest rates, with large quantitative easing, suggests that core monetary doctrines are wrong. It suggests that inflation can be stable and determinate under a nominal interest rate peg, and that arbitrary amounts of interest-paying reserves are not inflationary. Of the known alternatives, only the new-Keynesian model merged with the fiscal theory of the price level is consistent with this simple interpretation of the facts.

I explore two implications of this conclusion. First, what happens if central banks raise interest rates? Inflation stability suggests that higher nominal interest rates will result in higher long-run inflation. But can higher interest rates temporarily reduce inflation? Yes, but only by a novel mechanism that depends crucially on fiscal policy. Second, what are the implications for the stance of monetary policy and the urgency to "normalize?" Inflation stability implies that low-interest rate monetary policy is, perhaps unintentionally, benign, producing a stable Friedman-optimal quantity of money, that a large interest-paying balance sheet can be maintained indefinitely, and that it might not be wise for central bankers to exploit a temporary negative inflation effect.

The fiscal anchoring required by this interpretation of the data responds to discount rates, however, and may not be as strong as it appears.

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1 Summary and overview

For nearly a decade in US, UK, and Europe, and three decades in Japan, short-term interest rates have been near zero, and thus stopped moving more than one-for-one with inflation as prescribed by the Taylor principle. In the last decade, central banks also embarked on immense open market operations. US quantitative easing (QE) raised bank reserves from \$50 billion to \$3,000 billion, a factor of 60.

The response to this important experiment in monetary policy has been silence. Inflation is stable, and if anything less volatile than before. There is no visually apparent difference in macroeconomic dynamics in the near-zero-rate and large-reserves state than before.

Existing theories of inflation make sharp predictions about this circumstance: Old-Keynesian models, characterized by adaptive expectations, and in use throughout the policy world, predict that inflation is unstable under passive ($i_t = \phi\pi_t + v_t^i;\ \phi < 1$) monetary policy, and therefore predict a deflation spiral at the zero bound. It did not happen. Monetarist thought, $MV = PY$ with V "stable" in the long run, predicts that a massive increase in reserves must lead to galloping inflation. It did not happen.

New-Keynesian models, featuring rational expectations and interest rate targets, predict that inflation is stable under passive policy. Unless one adds frictions, those models also predict that quantitative easing operations are irrelevant. The observed inflation stability is thus a big feather in the new-Keynesian cap. But standard new-Keynesian models predict that inflation becomes indeterminate when interest rates cannot or do not move, and so suffers from additional sunspot volatility. These models also predict a menagerie of policy paradoxes: Productivity improvements are bad, promises further in the future have larger effects today, and reducing price stickiness makes matters worse, without limit.

This is a Michelson-Morley[1] moment for monetary policy. We observe a decisive experiment, in which previously hard-to-distinguish theories clearly predict large out-

[1] In 1887, Albert A. Michelson and Edward W. Morley set out to measure the speed of Earth through the ether, the substance thought in its day to carry light waves, by measuring the difference between the speed of light in various directions. They found nothing: the speed of light is the same in all directions, and the Earth appears to be still. Special relativity follows pretty much from this observation alone.

comes. That experiment yields a null result, which cleanly invalidates those theories.

Now, any theory, especially in economics, invites rescue by epicycles. Perhaps inflation really is unstable, but artful quantitative easing just offset the deflation vortex, Or perhaps wages are much “stickier” than we thought, or money is taking a long time to leak from reserves to broader aggregates, so we just need to wait a bit more for unstable inflation to show itself. Perhaps a peg really does lead to indeterminacy and sunspots, but expectations about active ($\phi > 1$) monetary policy in the far future takes the place of current Taylor rule responses to select equilibria. Perhaps the Earth drags the ether along with it.

Occam responds: Perhaps. Or, perhaps one should take seriously the simplest answer: Perhaps inflation can be stable and determinate under passive monetary policy, including an interest rate peg, and with arbitrarily large interest-bearing reserves. Classic contrary doctrines were simply wrong.

We are not left, as Michelson and Morley were, with a puzzle – a set of facts that existing theories cannot account for. Adding the fiscal theory of the price level to to the standard rational-expectations framework, including new-Keynesian price stickiness, we obtain a simple economic model in which inflation can be stable and determinate under passive policy, zero bound, or even a peg, and despite arbitrary quantitative easing. The model also has a smooth frictionless limit, and resolves new-Keynesian policy paradoxes.

What does this experience, and theoretical interpretation, imply about monetary policy going forward?

First, if inflation is stable under an interest rate peg, then it would seem to follow that were the central bank to raise interest rates and leave them there, then inflation must eventually *rise*. This reversal of the usual sign of monetary policy has become known as the “neo-Fisherian” hypothesis.

However, higher interest rates might still *temporarily* lower inflation before eventually raising it. I investigate what minimal set of ingredients it takes to produce a negative short-run impact of interest rates on inflation.

This quest has a larger goal. We do not have a simple economic baseline model that produces a negative response of inflation to a rise in interest rates, in our world of interest rate targets and abundant excess reserves. If there is a short-run negative relationship, what is its basic economic nature?

The natural starting place in this quest is the simple frictionless Fisherian model, $i_t = r + E_t\pi_{t+1}$. A rise in interest rates i produces an immediate and permanent rise in expected inflation. In the search for a temporary negative sign I add to this basic frictionless model 1) new-Keynesian pricing frictions 2) backwards-looking Phillips curves 3) monetary frictions. These ingredients robustly fail to produce the short-run negative sign. You cannot truthfully explain, say, to an undergraduate or policy maker that higher interest rates produce lower inflation because prices are sticky, or because lower money supply drives up rates and down prices, and our fancy models build on this basic intuition.

One ingredient can robustly and simply produce the desired temporary negative sign. If we add long-term debt, and if we assume that fiscal policy does not respond to variation in the real cost of debt, then a rise in interest rates can produce a temporary decline in inflation. Higher nominal rates lower the nominal present value of long-term debt; absent any change in expected surpluses, the price level must fall to restore the real present value of the debt. That works, but it is a rather dramatically novel mechanism relative to all standard economic stories and policy discussion.

We are left with a logical conundrum: Either 1) The world really is Fisherian, higher interest rates raise inflation in both short and long run; 2) more complex ingredients, including frictions or irrationalities, are *necessary* as well as sufficient to deliver the negative sign, so this hallowed belief relies on those complex ingredients; 3) the negative sign ultimately relies on the fiscal theory story involving long-term debt – and has nothing to do with any of the mechanisms commonly alluded for it.

The first view is not as crazy as it seems. The VAR evidence for the traditional sign, reviewed below, is weak. Perhaps the persistent “price puzzle” was trying to tell us something for all these decades. Raw correlations are of little use, as interest rates and inflation move closely together under either theoretical view, at least away from the zero bound.

The second set of policy issues: Is it important for central banks to raise expected inflation, raise nominal rates, reduce the size of their balance sheets, ration non-interest-bearing reserves, and return to active Taylor rules? (The Fed refers to this package, roughly what it was doing 1982-2007, as "normalizing" policy, though it's not clear that the package deserves the normative flavor of that blessing, or that alternatives deserve the implied "abnormal" status.)

The experience of stable inflation at near-zero interest rates suggests that we *can* instead live the Friedman optimal quantity of money forever – at least a large balance sheet of interest-bearing reserves financed by short-term government debt, potentially low or zero rates with corresponding low inflation or even slight deflation, and consequently permanent abandonment of the Taylor principle. That conclusion requires verification in theory – at least the existence of a theory that argues current experience can continue, and examination of its auxiliary assumptions, which I provide.

Whether we *should* do so requires listing (here) and quantifying (eventually) trade-offs – the ability of a stochastic peg to accommodate adverse "natural rate" shocks without violating the lower bound, the advantages of the Friedman optimal quantity of money, the practicality of exploiting a poorly understood temporary negative sign.

Finally, stable but temporary negative reaction is a quite different beast than unstable. Even if we find a model that is stable and determinate, but produces a negative reaction of inflation to expected interest rate rises, that remains quite different from restoring the classic view of instability and a permanent negative effect. In particular, the former model does not imply Taylor-rule style stabilizations. The Taylor rule is wise for an unstable model (old-Keynesian) or an indeterminate model (new-Keynesian).

I address a few common objections. How can the fiscal theory be consistent with low inflation, given huge debts and ongoing deficits? Fortunately, the fiscal theory does not predict a tight linkage between current debts, deficits and inflation. Discount rates matter as well, and discount rates for government debt are very low. What about other pegs, which did fall apart? Answer: fiscal policy fell apart.

That last observation leads to a final warning. My careful hedging, that an interest rate peg *can* be stable, refers to the necessary fiscal foundations. If fiscal foundations evaporate, that theory warns, and harsh experience reminds us, so can our benign moment of subdued and quiet inflation. I sketch the mechanism.

2 Nothing happened

Figure 1 presents the last 20 years of interest rates, inflation and reserves in the U.S. The

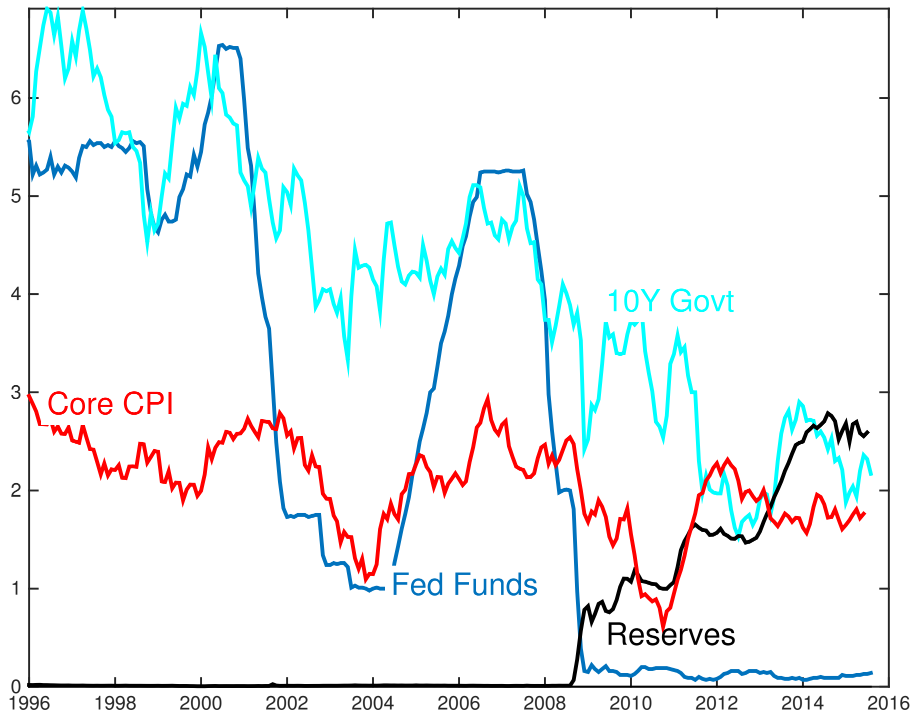


Figure 1: Recent US experience.

federal funds rate follows its familiar cyclical pattern, until it hits essentially zero in 2008 and stays there. In 2008-2009, the severity of the recession and low inflation required sharply negative interest rates, in most observers' eyes and in most specifications of a Taylor rule. The "zero bound" was binding. If you see data only up to the bottom of inflation in late 2010, and if you view inflation as unstable under passive monetary policy, fear of a "deflation spiral" is natural and justified.

But it never happened. Despite interest rates stuck at zero, inflation rebounded with about the same pattern as it did following the previous two much milder recessions.

Interest rates have remained near zero ever since. The interpretation of this period is less obvious. To many, we still are in a period in which the "natural" rate is low, the zero bound binds, and consequently a deflation spiral still looms. To others, the Taylor rule recommended interest rates to rise by about 2012, so we should regard near-zero rates as a policy choice, not a bound – one that threatens inflation, not deflation. But interest rates are stuck either way. (Since inflation is low, the difference in views comes down to one's views about the desirable response of inflation to output gaps, ϕ_y in $i_t = \phi_\pi \pi_t + \phi_y y_t + v_t^i$, and views on how big the output gap is.)

Still, nothing happened. Inflation gently declined. Overall, inflation is dominated by a slow 20 year downward trend. On top of that trend there is the usual business cycle pattern, that inflation declines in a recession and then rises again when the recession is over.

Moreover, to the unaided eye, inflation dynamics are unaffected by the long period of immobile interest rates. In fact, after 2012, when the financial crisis and deep recession receded, inflation volatility is lower than it was before 2009, when interest rates could "actively" stabilize inflation. A theory in which $\phi > 1$ vs $\phi = 0$ is an important state variable for stability, determinacy, or other economic dynamics is challenged by this period.

The Fed also increased bank reserves, from about \$50 billion to nearly \$3,000 billion, in three quantitative easing (QE) operations, as shown in Figure 1. Once again, nothing visible happened. QE2 is associated with a rise in inflation, but QE1 and QE3 are associated with a decline. And the rise in inflation coincident with QE2 mirrors the QE-free rise coming out of the much milder 2001 recessions. QE2 and QE3 were supposed to lower long-term interest rates. To the eye, the 20 year downward trend in long term rates is essentially unaffected by QE. If anything, long-term interest rates rose coincident with QE operations.

Figure 2 plots the unemployment rate and GDP growth rate. Together with Figure 1, these figures also show no visible difference in macroeconomic dynamics in and out of the zero rate / QE state. Yes, there was a bigger shock in 2008. But the unemployment recovery looks if anything a bit faster than previous recessions. Output growth, though too low in most opinions, is if anything less volatile than before.

Figure 3 tells a similar but longer story for Japan. Japanese interest rates declined swiftly in the early 1990s, and essentially hit zero in 1995. Again, armed with the traditional theory that inflation is unstable unless interest rates can move more than one-for-one in response, and seeing data up to the bottom of inflation in 2001, or again

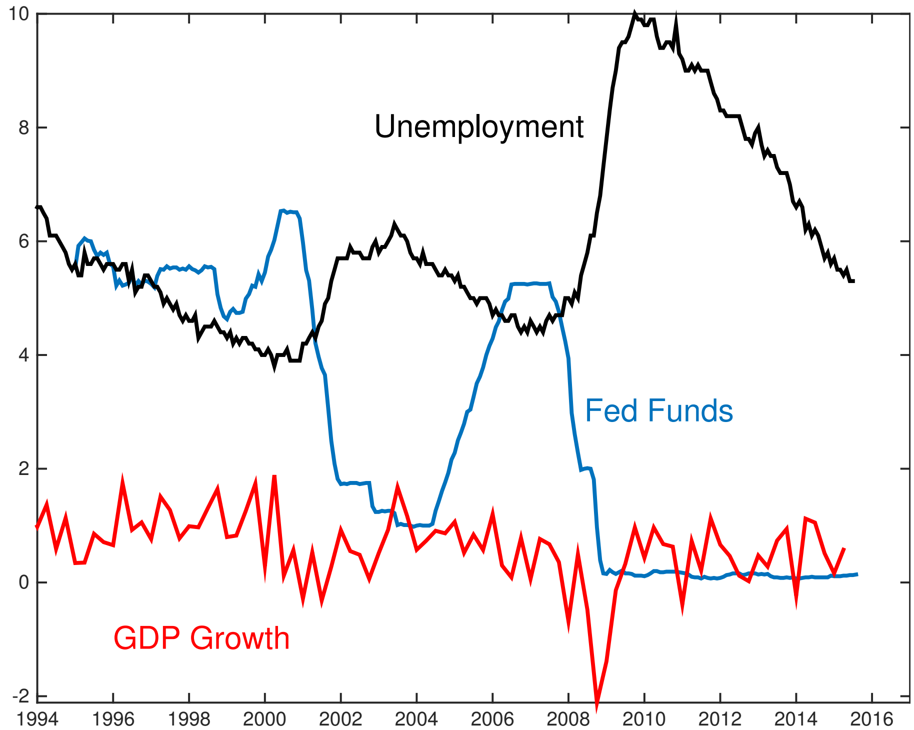


Figure 2: US federal funds rate, unemployment rate, and real GDP growth rate.

in late 2010, predicting a deflation "spiral" is natural and justified. But again, it never happened. Despite large fiscal stimulus and quantitative easing operations, Japanese interest rates stuck at zero with slight deflation for nearly two decades. The 10 year government bond rate never budged from its steady downward trend.

The bottom panel of Figure 3 repeats the story for Europe. Here the spread of low rates and slight deflation is even stronger than in the US.

Both Japan and Europe diverge from the U.S. in the last few years, with less inflation and lower interest rates. But are Japanese and European inflation lower despite their lower or even negative interest rates, or because of them?

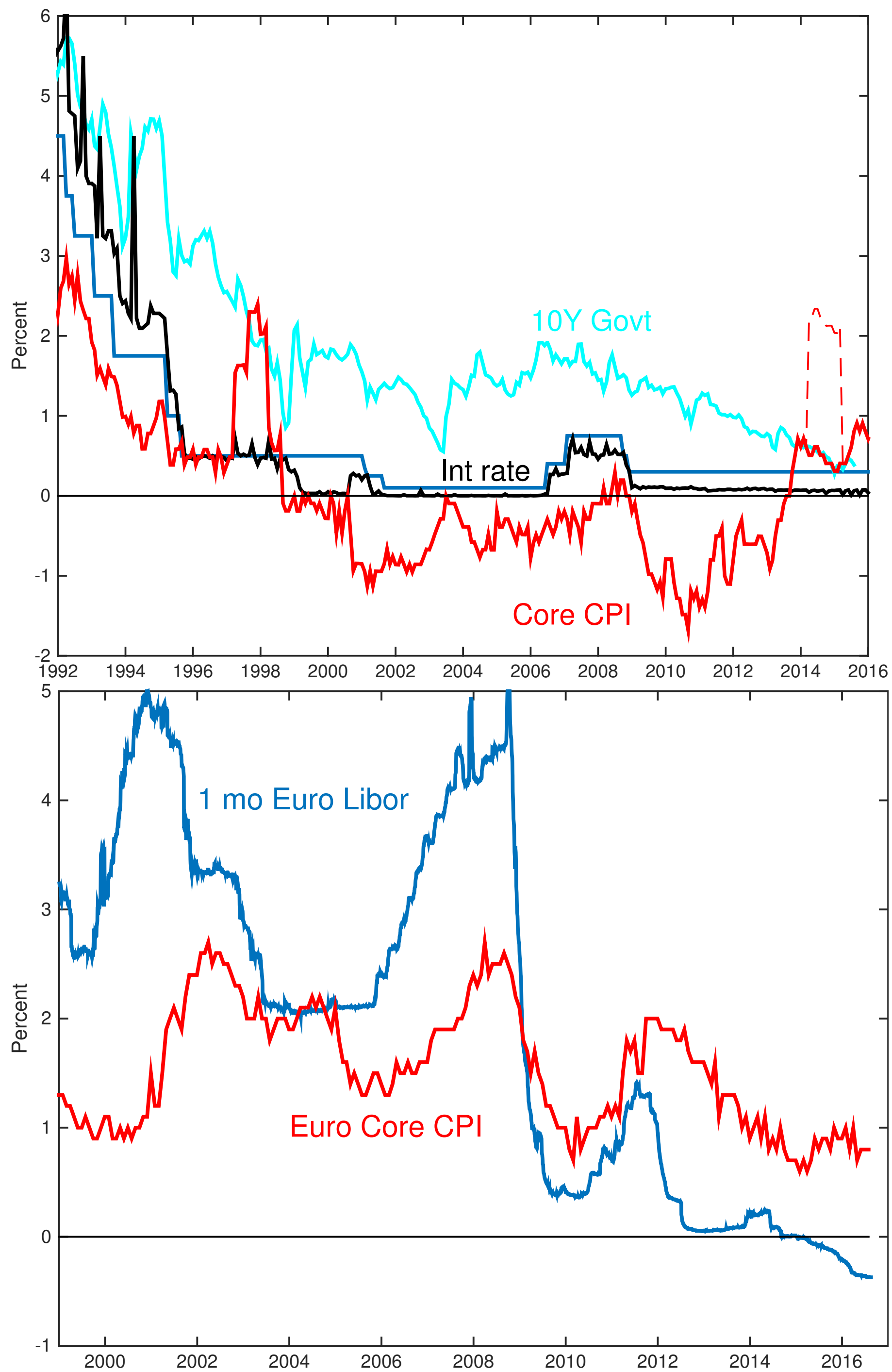


Figure 3: Japan and Europe. Top: Discount rate, Call rate, Core CPI, and 10 year Government Bond Yield in Japan. The thin presents the raw CPI data. Thick line adjusts the CPI for the consumption tax by forcing the April 2014 CPI rise to equal the rise in March 2014. Bottom: Europe

3 Theories

Old-Keynesian models predict that passive policy including an interest rate peg is unstable, and the Taylor rule stabilizes an otherwise unstable economy. In that model, the central bank must first lower interest rates to get inflation going, then quickly catch up to keep inflation from spiraling out of control. New-Keynesian models predict that passive policy including an interest rate peg is stable, but leave indeterminacies. The Taylor rule destabilizes an otherwise stable economy to remove local indeterminacies. In response to a permanent interest rate increase, this model predicts inflation will eventually rise, but the sign of the immediate response can go either way.

The key distinction between the models is rational vs. adaptive expectations. Since this is review of well-known results, my contribution here is a model that maximizes simplicity rather than realism or generality.

A visual metaphor may help to map out the purpose of the equations. Imagine a Fed chair balancing a walking stick upside down. Metaphorically, the end of the stick she holds is the interest rate, and the other end is inflation. The stick wants to fall over – inflation is unstable. But the Fed chair can keep inflation steady if she quickly moves the bottom of the stick more than one for one with little movements in the top, balancing it as a seal balances a ball on its nose. The Taylor principle stabilizes the dynamic system. If she wants more inflation, she must first move the bottom stick slightly the wrong way, get the top going, and then swiftly catch up.

Suppose instead our Fed chair holds her walking stick right side up. Now the stick is stable. Even just holding the top, the bottom will swing back in to place. To get the stick going, she need only move the top in the direction she wants to go. Still, however, the bottom of the stick may go the opposite way temporarily if she moves the top quickly. (I can't do determinacy in this metaphor alas.)

How could we tell from data on the position of the top and bottom of the stick which way it is being held? In normal times it would be very hard to do. The top and bottom of the stick would move together at low frequencies, and sometimes moves in opposite directions at high frequencies. But if someone stopped the Fed chair's hand, we would quickly sort it out: If the stick were being held from the bottom, unstable, it would topple over. If it were held from the top, stable, the inflation end would settle down to follow the fed Chair's hand.

3.1 A very simple model

Consider a Fisher equation, a Phillips curve, a static IS curve, and a Taylor rule for monetary policy:

$$i_t = r_t + \pi_t^e \qquad \text{Fisher} \tag{1}$$

$$\pi_t = \pi_t^e + \kappa x_t \qquad \text{Phillips} \tag{2}$$

$$x_t = -\sigma(r_t - v_t^r) \qquad \text{IS} \tag{3}$$

$$i_t = \phi \pi_t + v_t^i \qquad \text{Taylor} \tag{4}$$

where i is the nominal interest rate, r is the real interest rate, π is inflation, π^e is expected inflation, x is the output gap, v^i is a monetary policy disturbance, and we can label v^r as a "natural rate" disturbance, the latter two potentially serially correlated.

By specifying a static IS equation, without the usual term $E_t x_{t+1}$ on the right hand side, we can solve the model trivially without matrices or eigenvalues. The same points hold in more general and realistic models. Eliminating x and r, equating the resulting two expressions for the nominal interest rate i, we reduce the model to the solution of a single equation in π:

$$\phi \pi_t + v_t^i = -\frac{1}{\sigma\kappa}\pi_t + \left(1 + \frac{1}{\kappa\sigma}\right)\pi_t^e + v_t^r \; (= i_t) \tag{5}$$

The right-hand term of equation (5) is a Fisher equation. $i = \pi = \pi^e$ is a steady state. But the dynamics are smeared out in time due to price stickiness. Those dynamics are the key.

3.1.1 Old-Keynesian

Old-Keynesian models specify adaptive expectations, $\pi_t^e = \pi_{t-1}$. Substituting that specification in (5) we obtain

$$\pi_t = \frac{1+\sigma\kappa}{1+\phi\sigma\kappa}\pi_{t-1} - \frac{\sigma\kappa}{1+\phi\sigma\kappa}(v_t^i - v_t^r) \tag{6}$$

For $\phi < 1$, and at a peg $\phi = 0$ in particular, the dynamics of this system are *unstable* and *determinate*. The coefficient on lagged inflation is above one. There is only one solution.

The coefficient on the disturbance term is negative. Therefore, higher interest rates or a lower natural rate send inflation spiraling down, as Figure 4 illustrates.

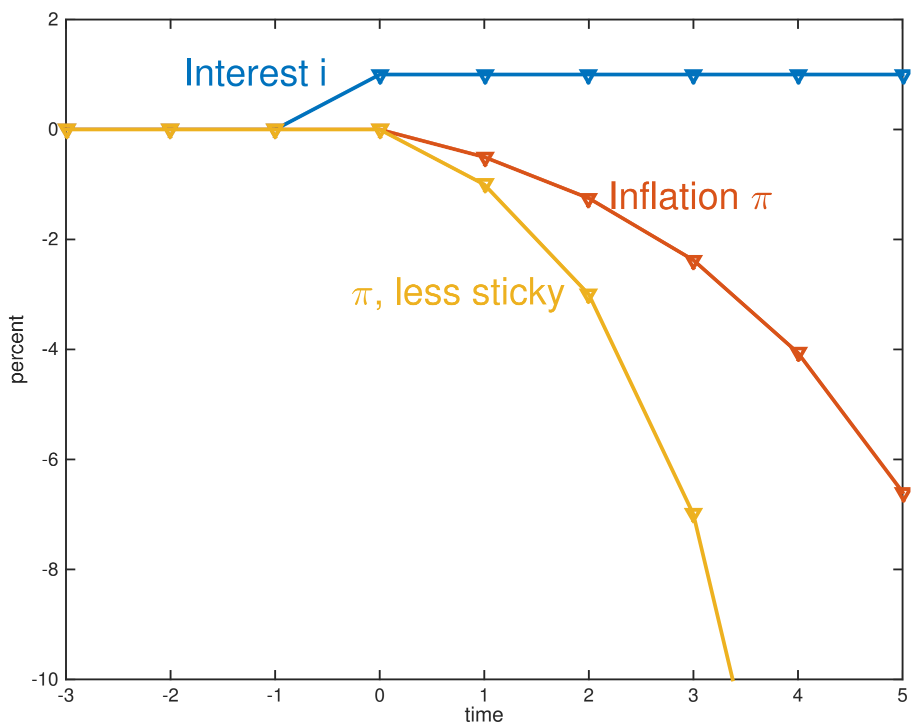


Figure 4: Simulation of a permanent interest rate rise or natural rate fall in the simple old-Keynesian model, with passive $\phi = 0$ policy. The baseline uses $\kappa = 1/2, \sigma = 1$. The "less sticky" case uses $\kappa = 1$.

In this model the Taylor rule *stabilizes* an otherwise unstable system. If $\phi > 1$, the coefficient on lagged inflation is less than one. A monetary contraction now has a uniformly negative, temporary (if v^i is temporary), but bounded effect on inflation.

But when $\phi < 1$, as it must be at the zero bound, then the model reverts to unstable dynamics. This model makes a clear prediction: At the zero bound, deflation must spiral.

Figure 4 includes the response for less price stickiness, $\kappa = 1$ instead of $\kappa = 1/2$. Sensibly, less sticky prices speed up dynamics. But that just makes the explosion happen faster, without limit. The adaptive expectations model does not approach the frictionless rational expectations limit in which interest rates rise, and expected inflation rises contemporaneously.

In the old-Keynesian view, we do not see exploding inflation, the opposite of Figure 4, because central banks aren't dumb enough to keep interest rates constant in the face of inflation.

To illustrate, Figure 5 plots the response of this very simple old-Keynesian model to a permanent monetary policy *shock* v_t^i.

Here, the tightening sends inflation uniformly down. Interest rates rise at first, to get disinflation going, but then must quickly follow inflation in order to stop it from going too far. This graph embodies exactly the sequence of events Friedman (1968) (p. 6) described of an interest rate change.

The distinction between monetary policy disturbance v_t^i and the path of observed interest rates i_t is important. A positive policy shock leads quickly to lower interest rates.

In the long run, interest rates embody the Fisher relationship, and interest rates move one for one with inflation. The model is not Fisherian, however, as it is unstable and interest rates must initially go the opposite way.

Figure 5 helps to illustrate why it is hard to tell an unstable model, whose central bank is following active policies and not letting instabilities erupt, from a stable model. Equilibrium interest rates and inflation will track up and down together in both cases. Until the interest rate cannot move. That's why the recent episode in which interest rates cannot move downward and did not move upward is so important.

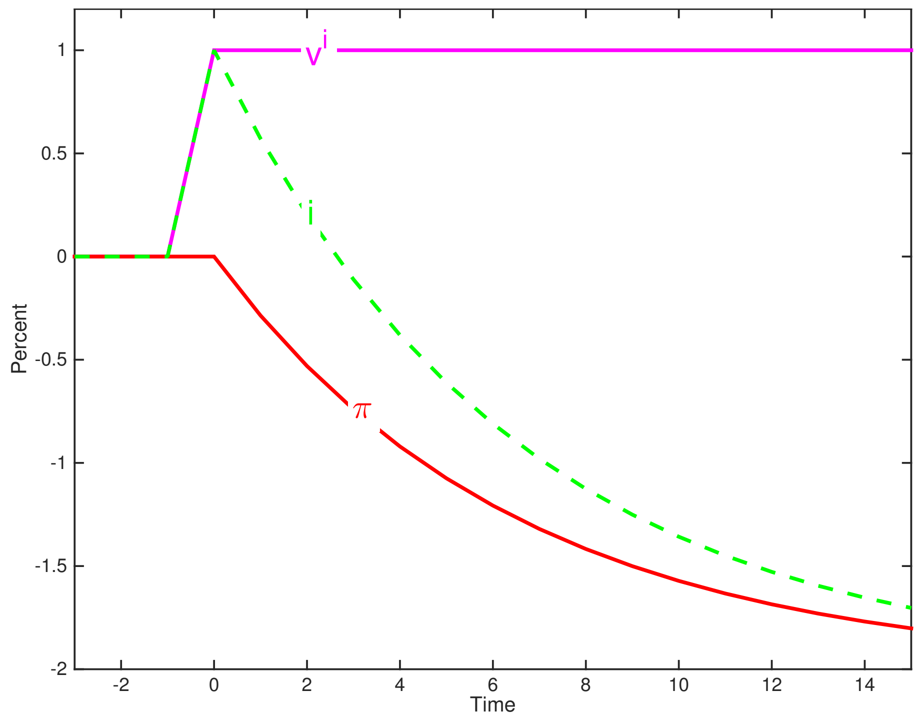


Figure 5: Response to a step-function monetary policy disturbance in the simple old-Keynesian model. The model is $x_t = -\sigma(i_t - \pi_{t-1})$; $\pi_t = \pi_{t-1} + \kappa x - t$; $i_t = \phi\pi_t + v_t^i$. The figure plots the response to a permanent 1% increase in v_t^i. $\phi = 1.5$, $\kappa = 0.2$, $\sigma = 1$.

3.1.2 New Keynesian

The new Keynesian tradition instead uses rational expectations: $\pi_t^e = E_t\pi_{t+1}$. Substituting this specification into (5), we obtain

$$E_t\pi_{t+1} = \frac{1+\phi\sigma\kappa}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa}(v_t^i - v_t^r). \tag{7}$$

For $\phi < 1$, the coefficient on π_t is less than one, so this model is *stable* all on its own, even under an interest rate peg $\phi = 0$. Adaptive, backward-looking expectations make price dynamics unstable, like driving a car by looking in the rear-view mirror. Rational, forward-looking expectations make price dynamics stable, as when drivers look forward and veer back on the road without outside help.

The last term in (7) is positive, so in this model, a monetary policy tightening v^i or a natural rate fall v^r *raise* inflation. Figure 6 illustrates the response of inflation to an interest rate rise in this case.

Already, the new-Keynesian model reverses the hallowed doctrine that interest rate pegs are unstable, and the widespread presumption that higher interest rates lower inflation.

A temporary fall in the natural rate v^r under an interest rate peg likewise raises inflation, and then inflation gradually declines as the natural rate reverts to normal. Inflation accommodates needed changes in the natural rate, albeit slowly, all by itself without the need for active Fed action or announcements. One might read the history of slowly decreasing inflation during recovery at the zero bound as an instance of this mechanism.

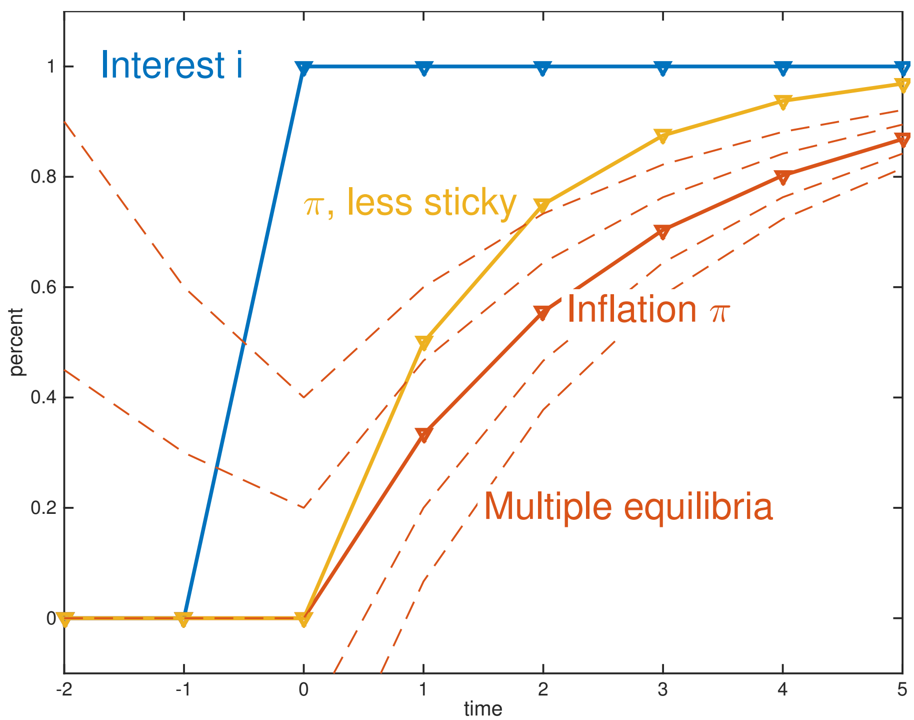


Figure 6: Simulation of an interest rate rise or natural rate fall in the simple new-Keynesian model with passive $\phi = 0$ monetary policy. The baseline uses $\kappa = 1/2, \sigma = 1$, the "less sticky" case uses $\kappa = 1$. Dashed lines indicate potential multiple equilibria.

This model with $\phi < 1$ is, however *indeterminate*. It only ties down expected inflation $E_t\pi_{t+1}$, where the old-Keynesian model ties down actual inflation. To the solutions of this model we can add any expectational error, δ_{t+1}, such that $E_t\delta_{t+1} = 0$, and then write the model's solutions as

$$\pi_{t+1} = \frac{1+\sigma\kappa\phi}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa}(v_t^i - v_t^r) + \delta_{t+1}. \tag{8}$$

The δ shocks that index multiple equilibria are "sunspots." In the usual causal interpretation of the equations, small changes in expectations about the future $E_t\pi_{t+j}$, perhaps coordinated by economically unimportant events such as sunspots – or Federal Reserve officials' speeches – induce jumps between equilibria π_t. I indicate such multiple equilibria by thin dashed lines in Figure 6. The model can jump between any of these, and then the dashed line indicates expected inflation after such a jump. Sunspots can induce additional inflation volatility.

One might restore the belief that higher interest rates at least temporarily lower inflation by engineering a negative sunspot shock coincident with the interest rate rise. I explore these issues below.

In this model, a Taylor rule induces *instability* into an otherwise *stable* model in order to try to render it *determinate* – to select a particular choice of $\{\delta_{t+1}\}$. For $\phi > 1$, expected inflation blows up for all values of inflation π_t other than

$$\pi_t = -\sum_{j=0}^{\infty}\left(\frac{1+\sigma\kappa}{1+\sigma\kappa\phi}\right)^{j+1}\frac{\sigma\kappa}{1+\sigma\kappa}E_t(v_{t+j}^i - v_{t+j}^r). \tag{9}$$

Equivalently, taking $E_t - E_{t-1}$,

$$\delta_t = -\sum_{j=0}^{\infty}\left(\frac{1+\sigma\kappa}{1+\sigma\kappa\phi}\right)^{j+1}\frac{\sigma\kappa}{1+\sigma\kappa}\left(E_t - E_{t-1}\right)(v_{t+j}^i - v_{t+j}^r). \tag{10}$$

The economy jumps by an expectational error δ_t just enough so that expected inflation does not explode.

This method of inducing determinacy is not entirely uncontroversial (Cochrane (2011)). That controversy is one motivation to look for another theory. But that controversy is not central here. If, because of the zero bound or other reasons, the interest rate follows a peg or passive policy $\phi < 1$, we are back to the conclusion of (8): stability plus indeterminacy.

3.1.3 Fiscal theory of the price level

To show how the fiscal theory of the price level enters this kind of model in the simplest way, I specify one-period or floating-rate debt. Then the equation stating that the real value of nominal debt equals the present value of primary (net of interest) surpluses reads

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} M_{t,t+j} s_{t+j} = E_t \sum_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{11}$$

Here, B_{t-1} is the face value of one-period debt, issued at $t-1$ and coming due at t, P_t is the price level and s_t is the real primary surplus. In the first equality, we discount the future with a general stochastic discount factor M. In the second equality, we discount the future with the ex-post real rate of return on government debt. Either of these statements is valid in general; the latter ex-post as well. The third version specializes to a constant real interest rate.

In general, real interest rate variation affects the present value of surpluses on the right hand side of (11). Models with sticky prices imply variation in real interest rates. I will argue below that such real interest rate variation is of first-order importance to understand data, experience, and policy via the fiscal theory. However, the stability and determinacy points are not affected by long-term debt or real rate variation, so I specify constant real rates and short term debt to make basic points here, then generalize at a cost in algebra below.

Multiplying and dividing (11) by P_{t-1} and taking innovations,

$$\frac{B_{t-1}}{P_{t-1}} (E_t - E_{t-1}) \left(\frac{P_{t-1}}{P_t} \right) = (E_t - E_{t-1}) \sum_{j=0}^{\infty} \beta^j s_{t+j}. \tag{12}$$

In this simple setup, *unexpected inflation is determined entirely by innovations to the expected present value of surpluses.*

Indeterminacy is the inability of the standard New Keynesian model to nail down unexpected inflation with passive policy $\phi < 1$, because we can always add any unexpected shock δ to the solution. Equation (12) shows that the fiscal theory of the price level solves the indeterminacy problem. The present value of future expected surpluses provides the "anchoring" of inflation expectations widely perceived by policy-makers.

In sum, the fiscal theory of the price level merged with the new-Keynesian model says that with an interest rate peg, or passive $\phi < 1$ target, inflation is *stable*, and *determinate*.

"Fiscal theory" does not mean the central bank is powerless. Far from it. In this simple frictionless model, the Fed, by setting in interest rate target i_t will set expected inflation $i_t = r + E_t\pi_{t+1}$. The fiscal theory can then select which value of unexpected inflation $\pi_{t+1} - E_t\pi_{t+1}$ will occur. Unexpected inflation or disinflation revalues outstanding government debt, which requires changes in discounted future surpluses. Monetary policy – setting of interest rate targets – remains the central determinant of the path of expected inflation.

More concretely, multiplying and dividing (11) by P_{t-1} and taking expectations,

$$\frac{B_{t-1}}{P_{t-1}} E_{t-1}\left(\frac{P_{t-1}}{P_t}\right) = E_{t-1}\sum_{j=0}^{\infty}\beta^j s_{t+j}. \tag{13}$$

Together with

$$\frac{1}{1+i_{t-1}} = \beta E_{t-1}\frac{P_t}{P_t - 1},$$

we see that the government can set a nominal interest rate target, and vary that target without varying future surpluses. Doing so controls expected inflation. Equation (13) then describes the number of bonds that will be sold given the interest rate target and expected surpluses. This is important confirmation that the action is possible: an interest rate target will not lead to unbounded demand for bonds.

In the context of Figure 6, then, if we define "monetary policy" as changing nominal interest rates with no change in the present value of future surpluses, the fiscal theory gives us a completely money-free assurance that the Fed can target nominal interest rates without controlling surpluses, and the fiscal theory plus new-Keynesian model picks the $\delta = 0$ solid line of Figure 6 as the uniquely determined response to a monetary policy shock.

Figure 6 includes the case of less price stickiness, $\kappa = 1$ in place of $\kappa = 1/2$. Again, dynamics happen more quickly. But in this case, dynamics smoothly approach the frictionless limit, in which $i_t = r + E_t\pi_{t+1}$ and expected inflation rises immediately to match the rise in nominal interest rate.

3.2 Language

The language used to describe dynamic properties of economic models varies, and I can dispel some remaining confusion by being explicit about language.

I use the words “stable” and “unstable” in their classic engineering sense, to refer to the underlying dynamic system. A scalar system $z_{t+1} = Az_t + \varepsilon_{t+1}$ is stable if $|A| < 1$ and unstable if $|A| > 1$. Authors often use “stable” to mean the opposite of “volatile,” but they are distinct concepts. A stable system with large shocks can display lots of volatility.

I use the word “determinate” to mean that an economic model only has one equilibrium. Stability and determinacy are also distinct concepts, frequently confused.

A harder case concerns expectational models with roots greater than one, and a variable that can jump, $E_t(z_{t+1}) = Az_t + v_t$, $|A| > 1$. I continue to use the word “unstable” to describe their dynamics. However, if one rules out explosions and solves forward, $z_t = E_t \sum_{j=0}^{\infty} A^{-(j+1)} v_{t+j}$, then one could justifiably call the equilibrium path of $\{z_t\}$ “stable” since it always jumps just enough to forestall explosions, and samples show future z expected to revert back after a shock. This behavior is sometimes called “saddle-path stable.” I use the term “stationary” to describe this property of *equilibrium* paths, which is common to this case and to the stable case, using the word “stable” or “unstable” to describe the properties of the dynamic system, A. The “stability” of a forward-looking system with a jump variable is qualitatively different from that of a system with backward-looking dynamics and no jump variables.

The standard three-equation new-Keynesian model with passive $\phi < 1$ monetary policy has dynamics of the form $E_t(z_{t+1}) = Az_t + v_t$ in which one eigenvalue of A is greater than one and unstable, and the other is less than one and stable. One could call such a model “mixed,” or develop additional language to describe the relative number of unstable eigenvalues and expectational equations. However, in this case, we uncontroversially solve the unstable eigenvalue forward, determining one of output or inflation from the other or equivalently determining one eigenvector. Then, the other eigenvector follows a scalar $E_t(z_{t+1}) = Az_t + v_t$ with $A < 1$. I use the language "stable" vs "unstable" and "determinate" vs "indeterminate" to describe the remaining eigenvector.

The data speak fundamentally about these dynamic properties of models, not the model ingredients directly. A large class of old-Keynesian and monetarist models with adaptive expectations and passive $\phi < 1$ interest rate targets produce unstable, determinate dynamics. A large class of new-Keynesian models with rational expectations and passive interest rate targets produce stable, indeterminate dynamics, and adding the fiscal theory produces stable, determinate dynamics. But I do not claim that *all* models in these categories produce such dynamics.

4 Michelson-Morley and Occam

The simplest interpretation of the data presented in section 2 is that we are living at something approximating an interest rate peg, or at least a prolonged period of insensitivity to inflation $\phi < 1$, and that inflation is stable and has low volatility in this state. The data also suggest that reserves and short-term treasuries are essentially perfect substitutes when they pay the same interest rate, so an arbitrarily large balance sheet causes no inflation.

If we accept from this experience that our models should allow inflation to be stable and determinate at an interest rate peg, then only the new-Keynesian model with fiscal theory of the price level is left standing.

Theories fail publicly when they predict nothing and big things happen. This was the case in the 1970s, when unexpected stagflation broke out, and again in the early 1980s, when inflation dropped suddenly as well. Theories fail no less when they predict movements that do not happen. That is the case now. It's just less public

Of course, one can interpret economic data and apply models in many ways.

4.1 Offsetting instabilities?

Perhaps the economy really is unstable as in old-Keynesian/monetarist models, but inflationary quantitative easing and deftly-timed forward guidance just offset a zero-bound deflationary spiral, leading to the false impression of stability. An old-Keynesian Taylor rule Fed was already thought to stabilize an unstable economy. Perhaps the new QE and talk tools were even more effective at quashing unstable deflation than low interest rates, ruled out by the zero bound, would have been.

Perhaps. But when the night at sea was quiet Occam suggests that the weather was calm, rather than believe that the captain deftly steered the ship just close enough to the hurricane of hyperinflation to avoid the great whirlpool of deflation.

4.2 Really slow unstable dynamics?

Perhaps the economy really is unstable as in old-Keynesian/monetarist models, but the dynamics are much slower than we previously thought, and observed in previous episodes (under this interpretation) such as the 1970s, either due to new mechanisms or different parameters. In the extremely simple model I presented above, lower κ – stickier prices and wages – and lower σ – output less affected by real interest rate variation – both slow down dynamics. If so, the deflation spiral is still waiting to break out any day, even in Japan. Likewise, perhaps the "long-run stability" of velocity, even at low interest differentials, is much longer-run than previously thought; velocity will finally recover some day soon and inflation will break out.

Perhaps. But perhaps not. First, this speculation is all pretty clearly ex-post rationalization. The broad consensus of people using old-Keynesian policy models was and remains that a deflation spiral was a danger. Many monetarists did clearly expect quantitative easing to lead to runaway inflation.

This observation is praise, not criticism. The models clearly made those predictions. People should commended for offering the advice that their models present. The models were also broadly consistent with data, at least outside of Japan, before interest rates hit zero. In an unstable old-Keynesian model, when the Fed follows a $\phi > 1$ Taylor rule, equilibrium interest rates and inflation comove positively, driven by external shocks, just as the seal's nose and ball move together across the pool. Data from a period with varying interest rates don't distinguish stability vs. instability well. Similarly, MV=PY may describe how PY is determined from M control, or it may describe how M demand follows from P, Y, and V determined elsewhere. Until you push, it's hard to tell the two stories apart. That's why the zero bound / QE era is so revealing.

But now that we have pushed, and the models fail, we all understand the dangers of patching a model every time it fails.

Second, these sorts of patches to old-Keynesian models such as very "sticky" wages are either not worked out analytically, and compared with macroeconomic or microeconomic data – large job churn – more broadly. Old-Keynesian models dominate policy thinking but have vanished from academic journals, so the patches remains a suggestion, not a result. The large literature on wage stckiness in new-Keynesian models does not apply at all – as I hope is clear by now, new-Keynesian models are utterly different starting from basic properties such as stability and determinacy.

So, Occam suggests, perhaps not: perhaps the economy *is* stable at an interest rate peg.

4.3 Not really a peg? – Selection from future actions

Our interest rates are not exactly zero and not a true peg. I do not argue that we are at a peg – only that models which can credibly explain our experience must also have the property that inflation is stable and determinate at a true peg. But perhaps our experience is sufficiently different from a true peg that models which predict instability or indeterminacy at a true peg can survive.

"Zero bound" is not accurate, which is why I have avoided using the term. The Federal Funds rate has remained a few basis points above zero, and European interest rates have dipped slightly below zero. However, the main point for the models is whether the Federal Funds rate responds more than one for one to inflation or deflation, and that point does not require an exact zero bound.

The long period of immobile interest rates, and forward guidance that rates would stay low for a long time, is not a formal peg. There is really no such thing as a true peg –

even apparently permanent commitments such as a gold standard are suspended, devalued, or revoked in some circumstances. Our central bankers have long claimed that interest-rate stasis is one-sided, and they will raise interest rates if output and inflation rise – though so far Europe, Japan, and Sweden's attempts to raise rates were quickly abandoned. So, perhaps the difference between actual policy and an interest rate peg is large enough that our models need not display stability or determinacy at a peg in order to be consistent with experience.

How? In an active literature such as Werning (2012) or Eggertsson and Mehrotra (2014), expectations of possible future active, destabilizing, Taylor rules take the place of current Taylor principle responses to select equilibria while interest rates are zero in new-Keynesian models. In these papers, eventually, either deterministically or stochastically, we will leave the zero interest rate range. Modelers specify the destabilizing version of the Taylor rule to select a unique locally-bounded equilibrium around the inflation target in that future state. Modelers tie multiple equilibria during the zero-rate period to the following equilibria, and thereby eliminate indeterminacies during our apparent peg.

Here is a concrete example. From time $t = 0$ to $t = T$, there is a negative natural rate shock, $v_t^r = -2\%$. At time $t = T$ the natural rate shock passes so $v_t^r = 0$. The Fed follows a constrained active Taylor rule, so together interest rates and inflation follow

$$i_t = \max\left[\pi^* + \phi_\pi(\pi_t - \pi^*), 0\right] \tag{14}$$

$$\pi_t = (1 + \kappa\sigma)E_t\pi_{t+1} - \kappa\sigma(i_t - v_t^r) \tag{15}$$

Figure 7 shows the corresponding equilibria. The thick line in the middle is the selected equilibrium. It fairs in to the inflation target $\pi^* = 2\%$ at $t = T$. The alternative equilibria are selected even in the stable region, for $t < T$, by the fact that they diverge from the inflation target for $t > T$.

(In this simulation, I use the active $\phi = 2$ but zero-bound constrained form of the Taylor rule (14) throughout, even for $t < T$. As a result, equilibria which start with high enough inflation to free interest rates from the zero bound are already unstable at $t < T$. If one specifies a pure peg until $t = T$, then these equilibria are stable util $t = T$, and explode only afterwards. Werning (2012) and Cochrane (2014c) analyze this case.)

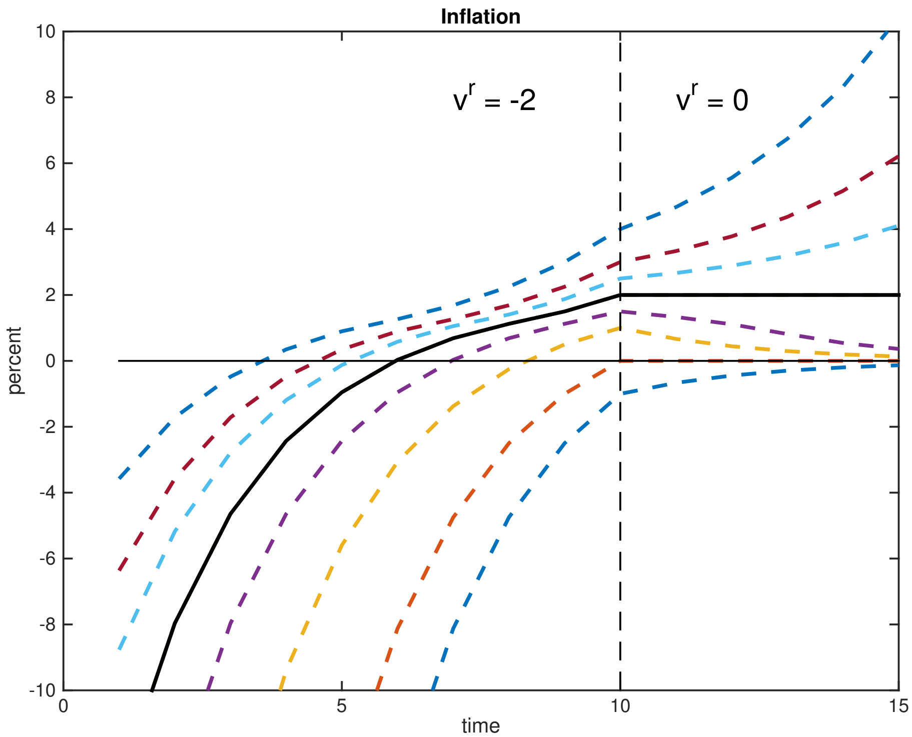
This equilibrium-selection scheme has many troubles. "Anchoring" of inflation expectations does not occur because the Fed is expected to *stabilize* inflation around the inflation target, but because the Fed is expected to *destabilize* inflation should it diverge from the target. More deeply, equilibria in which inflation udershoots the target return back to zero inflation and zero interest rates. They are *locally* unstable around the target π^* and thus locally determinate, but not *globally* unstable and thus globally indeterminate. The rationale for ruling them out is tenuous.

Figure 7 also illustrates a predictive failure of this model, highlighted by Werning (2012) and Cochrane (2014c). It predicts a jump to deflation at $t = 0$ when the shock hits, which then rapidly improves. This did not happen.

Figure 7 likewise illustrates some of the policy paradoxes highlighted by Werning (2012), Wieland (2015), Cochrane (2014c). Forward stable means backward unstable, so small change in the inflation target π^* on the right move the left end of equilibrium inflation around a lot. Forward guidance has large effects. Moreover, the further in the future the end of the shock and promise occurs, the larger the effect at time 0, and as price stickiness is reduced, the dynamics happen faster. Less price stickiness implies larger deflation and greater effect of such promises.

Here too, this anchoring story is also an ex-post patch to a theory, given pretty uniform expectation and analysis that the zero bound and passive policy must result in additional sunspot volatility, before it didn't.

Do people really pay that much attention to promises by Federal Reserve officials – and distinguish them from the routinely broken promises of other government functionaries – Treasury secretaries who routinely promise to end deficits one year after their president's term of office? Does all concrete action of monetary policy really vanish, leaving only expectations of far-future off-equilibrium threats behind? Can monetary policy really speak loudly with no stick? Did Japan really avoid deflation in 2001 because people expected some sort of explosive promises around a 2% inflation target to emerge and select in equilibria, maybe sometime in 2025 when Japan finally exits zero rates?



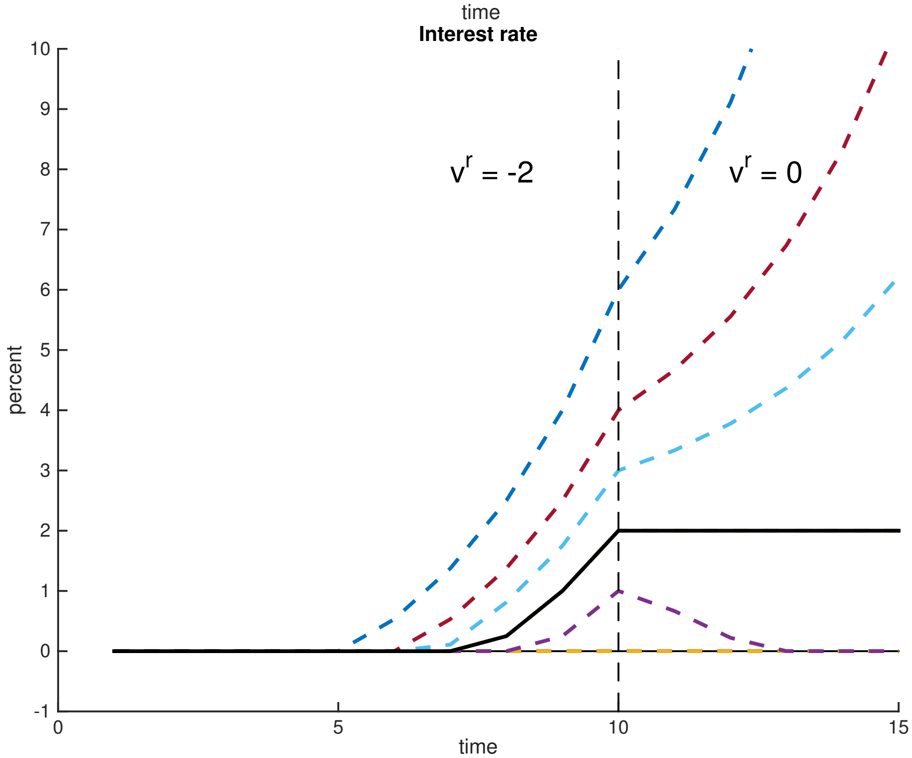


Figure 7: Selection by future and contingent Taylor rules. Top: Inflation. Bottom: Interest rates. Solid line is the selected equilibrium, dashed lines are alternative equilibria, indexed by inflation at time $t = T$. There is a natural rate shock $v^r = -2\%$ from time $t = 0$ to $t = T = 10$. The Fed follows a rule $i_t = \max\left[\pi^* + \phi_\pi(\pi_t - \pi^*), 0\right]$ and the simple new-Keynesian model reduces to $\pi_t = (1 + \kappa\sigma)E_t\pi_{t+1} - \kappa\sigma(i_t - v_t^r)$. $\sigma = 1$, $\kappa = 1/2$, $\phi = 2$.

Even Janet Yellen Yellen (2016) expresses a deep distrust that promises of future Taylor rules anchor inflation today:

> ...how does this anchoring process occur? Does a central bank have to keep actual inflation near the target rate for many years before inflation expectations completely conform? ...Or does ...a change in expectations requir[e] some combination of clear communications about policymakers' inflation goal, concrete policy actions,..., and at least some success in moving actual inflation toward its desired level ...?

Moreover, her language clearly states that anchoring results because a Taylor rule will, in the future, *stabilize* inflation around the target, in the old-Keynesian tradition, not *destabilize* inflation to produce determinacy as shown in Figure 7. If *she* doesn't believe the dynamics of Figure 7, why should we?

This search for far-future off-equilibrium expectations-anchoring might be more plausible if a much simpler solution were not at hand. The Fiscal theory picks one equilibrium of the stable new-Keynesian model directly. Expectations are anchored by the present value of future surpluses, not by what people believe the Fed might do to select equilibria after a jump to a higher-interest rate regime. Though stable, since it is unique, the fiscal-theory plus new-Keynesian model does not display policy paradoxes. Each equilibrium corresponds to an innovation in the present value of surpluses, and moving to a larger inflation at time t requires a larger change in the present value of surpluses at time t. The model also has a smooth frictionless limit. Occam's suggestion is pretty clear!

4.4 Sunspot volatility?

Perhaps the new-Keynesian prediction of higher inflation volatility under passive policy can be patched. The solution to the model with $\phi > 1$ is not the same as the solution with $\phi < 1$, plus some extra δ sunspot shocks. By (10), the variance of expectational shocks δ is not zero under active policy. Rather, there is a δ jump each period, just enough to offset that period's shocks. Therefore, it is possible for the determinate solutions of the model with $\phi > 1$ to exceed the volatility of the $\delta_t = 0$ solutions of the model with $\phi < 1$, so small sunspot shocks could still leave the model less volatile under passive than active policy.

As a concrete example, Appendix section 19.1 works out the volatility of inflation with AR(1) natural rate shocks $v_t^r = \rho v_{t-1}^r + \varepsilon_t^r$, in the active, forward-looking specification (9), and with $\phi = 0$ and $\delta_t = 0$ in the passive, backward-looking specification (8). For $\sigma\kappa = 1$, the ratio of the two inflation variances is

$$\frac{\text{backward } \sigma_\pi^2}{\text{forward } \sigma_\pi^2} = \frac{(2+\rho)}{(2-\rho)} \frac{\left[2(1-\rho) + (\phi - 1)\right]^2}{3}. \tag{16}$$

For $\phi = 1.5, \rho \gtrsim 0.5$ is sufficient for the ratio to be less than one. In the limit of very persistent shocks,

$$\lim_{\rho \to 1} \frac{\text{backward } \sigma_\pi^2}{\text{forward } \sigma_\pi^2} = (\phi - 1)^2$$

This forms a very simple counterexample: Passive policy – an interest rate peg – can display less inflation volatility than active policy, if shocks are persistent, the active parameter ϕ is not too large, and if sunspot shocks δ_t are not too large.

In this specification, higher values of the active parameter ϕ result in less inflation volatility. Optimal-policy more generally exercises point in that direction (Woodford (2003)). To the extent that the Federal Reserve followed good high-ϕ policy in the past, the chance of seeing lower volatility at the $\phi = 0$ bound is reduced. And even then, sunspots being ephemeral, we don't have much of a reason to limit their volatility. On the other hand, sunspots being ephemeral, "there weren't any sunspot shocks" is an irrefutable ex-post explanation of quiet. Phlogiston comes and goes as it pleases.

In any case, this is a novel possibility which as far as I know has not yet been advanced to rescue the prediction of higher inflation volatility at the zero bound. The new-Keynesian literature, summarizing larger and more realistic models, clearly warns that passive $\phi < 1$ monetary policy causes inflation volatility. That proposition is one of the model's most central empirical claims, explaining the greater volatility of the 1970s vs. the 1980s. If we throw out the prediction of higher volatility under passive policy in the 2010s, we are hard pressed not also to throw out the central prediction of higher volatility under passive policy in the 1970s.

For example, Clarida, Galí, and Gertler (2000), who found $\phi < 1$ in the 1970s, $\phi > 1$

in the 1980s, attribute the reduction of inflation volatility to that fact, writing (p. 149)

> ...the pre-Volcker rule leaves open the possibility of bursts of inflation and output that result from self-fulfilling changes in expectations. ... On the other hand, self-fulfilling fluctuations cannot occur under the estimated rule for the Volcker-Greenspan era since, within this regime, the Federal Reserve adjusts interest rates sufficiently to stabilize any changes in expected inflation.

Again on p. 177, they write

> Finally, we have argued that the pre-Volcker rule may have contained the seeds of macroeconomic instability that seemed to characterize the late sixties and seventies. In particular, in the context of a calibrated sticky price model, the pre-Volcker rule leaves open the possibility of bursts of inflation and output that result from self-fulfilling changes in expectations.

Benhabib, Schmitt-Grohé, and Uribe (2001) likewise write (p. 167)

> Perhaps the best-known result in this literature is that if fiscal solvency is preserved under all circumstances,[i.e. passive fiscal policy] ... a passive monetary policy, that is, a policy that underreacts to inflation by raising the nominal interest rate by less than the observed increase in inflation, destabilizes the economy by giving rise to expectations-driven fluctuations.

They write again in Benhabib, Schmitt-Grohé, and Uribe (2002), summarizing a "growing body of theoretical work,"

> Taylor rules contribute to aggregate stability because they guarantee the uniqueness of the rational expectations equilibrium, whereas interest rate feedback rules with an inflation coefficient of less than unity, also referred to as passive rules, are destabilizing because they render the equilibrium indeterminate, thus allowing for expectations-driven fluctuations.

(Both sets of authors use "stability" to mean "low volatility," not as I have used the term.) Benhabib, Schmitt-Grohé, and Uribe (2002) survey many other similar opinions, along with policy prescriptions to avoid the zero inflation state, all motivated by the prediction of extra volatility at that state.

Indeed, without extra sunspot volatility, low inflation is welfare-improving in this model. It is both Friedman-optimal, and reduces pricing distortions. Sunspot volatility is the main reason all of these authors have for the great effort to find policies that avoid the zero bound and return us to permanently higher inflation.

Occam says, let's take this wide reading of the model seriously. If, indeed, this model says that passive $\phi < 1$ policy gave rise to sunspot fluctuations and greater inflation volatility in the 1970s, while active $\phi > 1$ policy stabilized inflation by ruling out sunspot equilibria in the 1980s, if indeed Benhabib, Schmitt-Grohé, and Uribe (2002) and the rest of the iceberg of which they are the tip has spent 20 years since Japanese rates hit zero looking for ways out of an otherwise desirable circumstance because the models, broadly construed, predict additional inflation and output volatility at an interest rate peg, then we should take that prediction, and its failure, seriously.

The alternative, I suppose, is to patch up the model by finding some other feature of current policy that is eliminating sunspot shocks, but did not eliminate those shocks in the 1970s. Indeed, that is the feeling of much discussion that expectations are "anchored." But anchored by what? As above, the feeling that sooner or later interest rates must rise to strongly positive values, then the Fed will undertake active policy, and somehow avoid the theoretical problems of active policy, and people believe all this, seems strained. Just why didn't that work in the 1970s? The 1970s did not lack from forward-guidance promises by Federal Reserve officials! We even had those cute little WIN (whip inflation now) buttons. If anchoring was going to work this time, just why did researchers not know that fact, and loudly opine not to worry about the zero bound?

4.5 Occam summary

In sum, ex-post elaborations and modifications may be correct. We should learn, and adapt models as we gain more data. But that work is suspiciously complex, and clearly ex-post patches. One sniffs ether drag and epicycles, not Bayesian updates.

Occam suggests, perhaps not. Given that we have a simple, straightforward model, which neatly accounts for the facts without patching, and which is much cleaner theoretically – no paradoxical policies, and a smooth frictionless limit – perhaps it is instead the right answer. The only reason it's controversial, really, is just how deep the doctrines are that this model overturns: inflation can be stable at an interest rate peg.

The remaining doubt may be the plausibility of fiscal anchoring given large debts and deficits. That discussion comes below. The point here is theoretical simplicity.

5 Quantitative easing and monetarism

Quantitative easing and monetarism matter here for three reasons. First, is it possible that inflation really is unstable at a stuck interest rate, but the Fed's inflationary QE operations deftly offset a deflation spiral? (I'm not aware of a parallel argument that MV=PY somehow addressed indeterminacies, so I'll leave that out.)

Second, if so, is hyperinflation around the corner? To control inflation once it starts rising and interest rates leave the zero bound, must the Fed quickly return to the pre-2007 policy configuration with a much smaller balance sheet? Must it go further, and eliminate interest on reserves, returning to interest-rate management by rationing reserves rather than by varying the interest paid on reserves?

Third, the question has become a deep one: What is the fundamental determinant of inflation? Is inflation determined, fundamentally, from yesterday's inflation, an interest rate target and a Phillips curve (old-Keynesian)? Is inflation determined, fundamentally, from $\phi > 1$ and Fed off-equilibrium selection threats (new-Keynesian?) Is inflation determined, fundamentally from the fact that the government requires people to pay taxes with money (fiscal theory)? Is inflation determined, fundamentally, from MV=PY, a restricted supply of a special asset needed to make transactions? The last classic view needs an airing.

Quantitative easing has two parts: The Fed buys bonds or other assets, and issues reserves. Here I consider the question whether larger reserve supply is inflationary. In traditional monetarist thought, the M in MV=PY drives output Y and inflation P, and whether the M came from buying short-term Treasuries, long-term Treasuries, mortgage backed securities, or from buying nothing – from helicopter drops – makes no difference. Commenters who refer to QE not as "drying up bond markets," but as "injecting liquidity" echo monetarist thought.

Much of the current QE discussion takes on a diametrically opposite view: The liabilities (reserves) are irrelevant, but QE "works" by affecting segmented markets for the assets. It's the bonds, not the money. Since the economic issues are smaller – whether bond purchases affect interest rates by a few tens of basis points, not the sign and stability of inflation – I postpone that question. Complex QE – buying mortgage backed securities, long term bonds, or stocks – is equivalent to an open market operation of short-term debt for reserves plus a twist of long term debt, MBS, etc. for short term debt. I think here just about the traditional swap of short term debt for reserves, and leave the composition of the assets on the Fed's balance sheet for later.

Monetarist thought took a back seat during the interest-rate targeting period starting in 1982. However, when Japan hit zero interest rates in the 1990s, the idea came back quickly. Ben Bernanke advocated the view most prominently, among other alternative policies (see the review and fascinating discussion in Ball (2016)).

In simple terms, monetarists think about interest rate targets as just another way to control money supply. Facing $M^d(i)$, if you set i and let M^d determine M, or if you set M^s and let $M^s = M^d(i)$ doesn't really matter. An interest rate at zero is not particularly meaningful. That case means the Fed can no longer control money supply via an interest rate target. But nothing stops the Fed from printing up money directly, and letting MV=PY do its magic.

The behavior of velocity or money demand at zero interest rates is the stumbling block to this line of thought. Monetarist thought emphasizes the idea that velocity is "stable," at least in the "long run." Even at zero interest rates – or our current situation that reserves pay even more than Treasuries – even if velocity V decreases somewhat, it will soon bounce back and more M will lead to more PY.

The contrary view is that at zero interest rates, or zero interest costs due to interest on reserves, money and short-term bonds become perfect substitutes. Velocity becomes a correspondence, an upper bound, not a function of interest rates. MV = PY becomes V = PY/M. People are perfectly happy to hold reserves, directly or indirectly via bank accounts, in place of short-term treasuries. Open market operations have no more effect on spending than change operations, in which the Fed gives everyone two $10 bills for each $20.

Absent data, there really was no way to tell these views apart. Now there is, and the experiment is nearly as decisive as the stability of an interest rate peg.

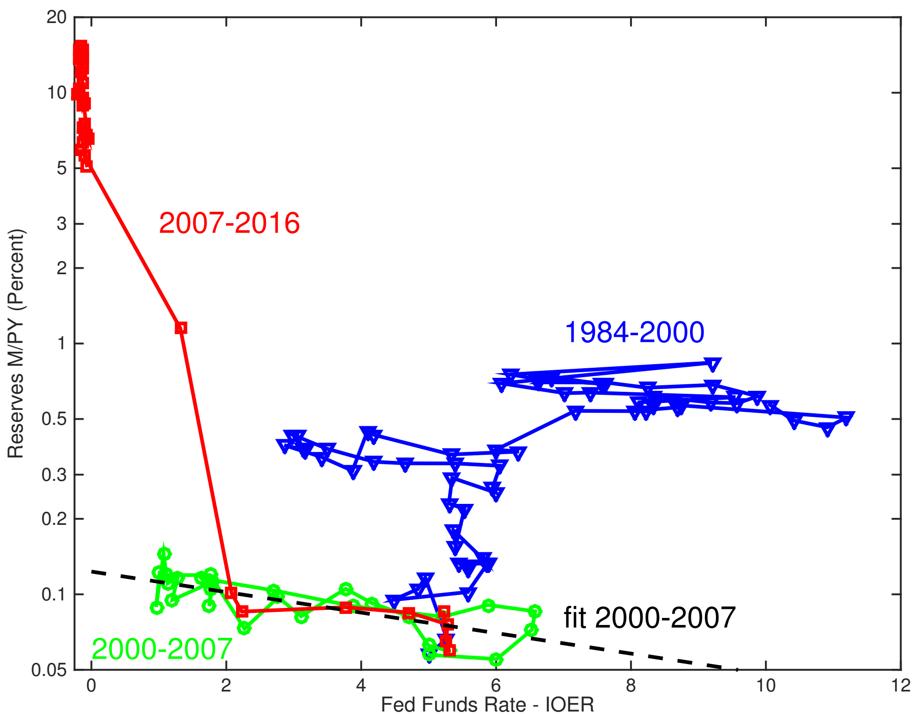


Figure 8: Reserves vs opportunity cost. Y axis: total reserves (Fred series WRESBAL) at end of quarter divided by nominal GDP; log scale. X axis: effective Federal funds rate (Fred FF) less interest on excess reserves (Federal Reserve website policy rates). Sample 1984:1-2016:2. Line fit by OLS 2000:1-2006:4

Figure 8 presents reserves, scaled by nominal GDP, as a function of their opportunity cost, the difference between the effective Federal Funds rate (the rate at which banks can lend out reserves) and the interest on excess reserves at the Fed. You see a steady decline in reserves from 1980 to 2000, as fewer bank liabilities required reserves and banks became better at avoiding excess reserves. You also see a negatively sloped curve in the periodic recessions. Following tradition, I'll just call this plot a "demand curve" without further ado. The dashed line is fit to the 2000-2007 period, and gives a conventional semi-elasticity log(reserves/PY) = constant - 0.094 (interest rate).

What does reserve demand do at zero opportunity cost? As Figure 8, shows, we have now run this out-of-sample experiment, on a grand scale – note the numbers on the log scale y axis. Reserves have increased by two orders of magnitude – from 0.1% of GDP to 15% of GDP – with no effect on inflation or nominal GDP.

The answer seems unavoidable: Reserve demand is a correspondence when reserves pay market interest rates; reserves and short-term debt are perfect substitutes; there is no tendency for velocity to revert to some "stable" previous value; there is no upper bound on the demand for "liquidity" at zero opportunity cost; arbitrary quantities of zero-cost reserves do not cause inflation.

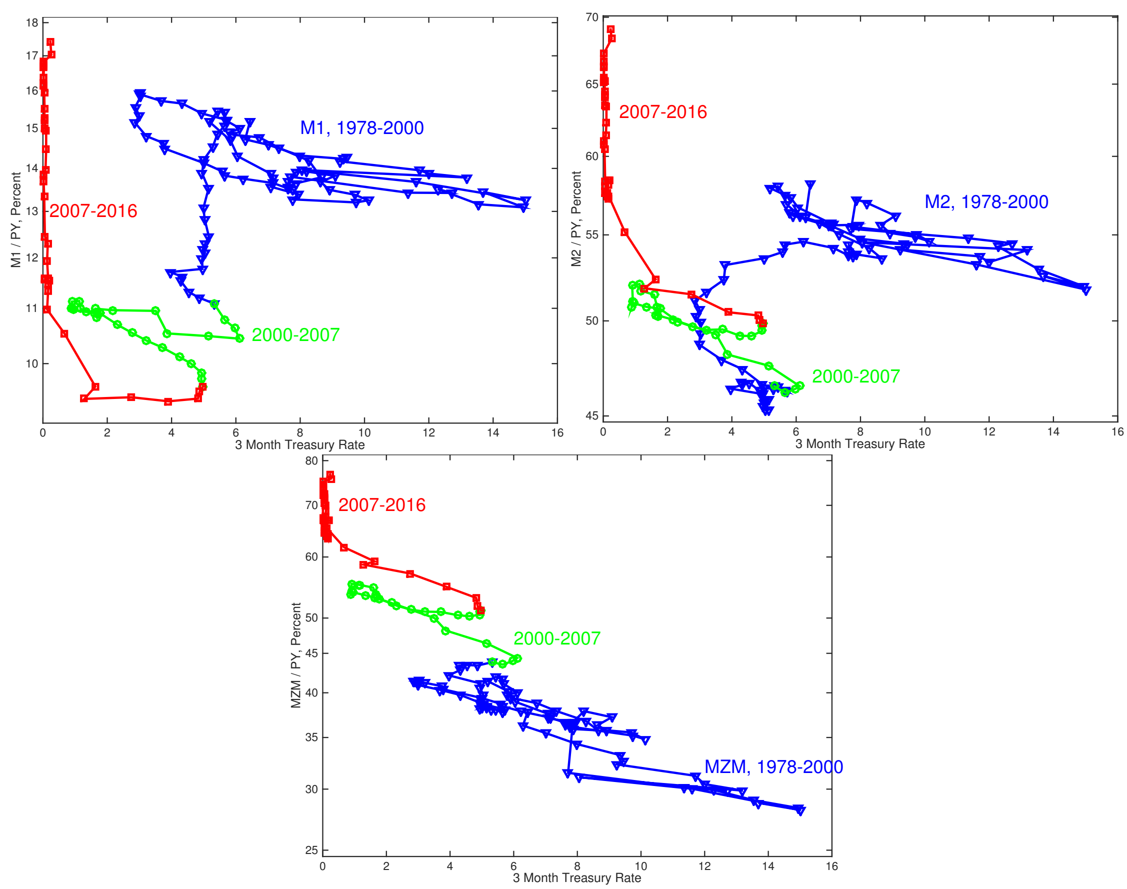


Figure 9: M/PY vs three month treasury bill rate, using M1, M2, and MZM.

One may object that reserves are not the relevant M in MV=PY. Figure 9 presents M1, M2, and MZM (M2 less small-denomination time deposits plus institutional money funds), as percentages of nominal GDP PY, versus the three-month Treasury bill rate. (Since many components of these aggregates pay interest, the three-month Treasury bill rate is not a good measure of their opportunity costs, but I'm both following tradition and keeping it simple.) Each aggregate has increased since 2007, but less, proportionally, than reserves have increased. M1 has increased from about 9% of GDP to almost 18%, a bit less than doubling – but not rising by a factor of 100. M2 has increased from 50% of GDP to almost 70% of GDP, "only" a 60% rise. Depending on what starting point you choose, MZM has risen the least, by 10 to 20 percentage points of GPD, or 20 to 40 percent overall.

These are still substantial increases, which if velocity were "stable" should result in equiproportionate rises in nominal income, and eventually the price level.

But even making these plots grants too much. How much inflation a "monetarist" model predicts for the recent period is beside the point. The point is whether arbitrary amounts of *reserves*, exchanged for short-term treasuries, cause any inflation. Even if one believes that M2 V = PY (say), and claims that a monetarist view does not predict inflation in the current period because reserves did not leak in to M2, that fact only emphasizes that arbitrary quantities of *reserves* are not inflationary, precisely because they did not leak into M2. That leakage is, in this view, a central part of the transmission mechanism. When banks are holding trillions of dollars of excess reserves, the money multiplier ceases to operate. Reserve requirements are an inequality constraint, not an equality constraint. So, to argue there is no inflation because M2 did not rise is precisely to admit that arbitrary quantities of interest-bearing reserves, corresponding to arbitrarily lower quantities of interest-bearing treasuries, are not inflationary.

Figure 8 really only makes a secondary point: What would happen if reserves were to leak to larger increases in M1, M2, or MZM? Figure 8 suggests that these aggregates display the same behavior as reserves, only on a smaller (so far) scale – they happily crawl up the vertical axis. There is nothing in their behavior so far to suggest that this correspondence could not reach the astonishing level that banks' willingness to hold reserves at the expense of treasuries have reached.

Another view admits that the conventional money multiplier is inoperative – reserves are so hugely beyond required reserves that bank money creation is unconstrained by reserves. However, in this view regulatory leverage and equity constraints still bind. Banks would create more money by lending out more, if only they weren't held to capital requirements or leverage requirements. This view requires some sort of upward sloping supply of capital, otherwise banks would retain earnings and undo the constraint. Nonetheless, the secondary interpretation of Figure 8 suggests that even then, were capital or leverage requirements loosened allowing banks to create more money, that creation would have little impact on inflation through the MV=PY channel.

Looking back at an 80 year controversy, one wonders why the "stability" of velocity, even at zero interest cost, and the perfect substitutability of treasuries for reserves, was so controversial. One answer may be that for most of that period, there was no coherent, simple, economic theory of the price level that could hold in that circumstance. In a monetarist world, strike MV=PY and nothing ties down P. Keynesian and new-Keynesian economics eventually had a Phillips curve to describe price dynamics, but no simple satisfactory alternative model of price level determination, at least to an economist longing for the transparency of MV=PY. So, it is natural to cling to the idea that velocity must be "stable," as otherwise the price level would be indeterminate. But now we have an equally simple theory – the fiscal theory – that ties down the price level when money and bonds are perfect substitutes, and a long period of apparent empirical validation. Economic beliefs survive for their usefulness as well as, or sometimes despite lack of, their theoretical consistency and empirical validation. This one has now lost the former, as well as latter, impulses.

My conclusion that abundant interest-bearing reserves will not cause inflation does not address many objections to the Fed's large balance sheet. The Fed's liabilities cause no problems for inflation, but its assets may cause problems. One may object to the Fed's purchases of long-term bonds, of mortgage-backed securities, and of other central bank's purchases of corporate bonds (ECB) and even stocks (BOJ), both on grounds that independent central banks should not try to influence directly risky asset prices, or on political economy grounds that such policies constitute credit allocation better done (if at all) by politically-accountable Treasuries. In this analysis, the Treasury could just as well supply fixed-value floating-rate electronically-transferable debt, i.e. reserves, and allow people and businesses, not just banks, to hold them. (For details on this proposal, see Cochrane (2015)). If the Treasury does so, the Fed itself would no longer need to act as the worlds' largest money market fund, transforming longer-term government debt into floating-rate government debt.

6 Do higher interest rates raise or lower inflation?

If we grant that inflation can be stable under an interest rate peg, that observation suggests that raising interest rates should sooner or later *raise* inflation, contrary to the usually presumed sign. If so, central banks will partly cause the inflation they wish to forestall – though in the event will likely congratulate themselves for their prescience. It also implies that current low inflation is in part due to pedal misapplication – central banks, by keeping interest rates low, partly caused the low inflation that they were trying, wisely or not, to prevent.

This empirical suggestion needs theoretical validation, beyond the too-simplified models of section 3. Do sensible models produce a long-run rise in inflation when interest rates rise? Then, if raising interest rates eventually leads to higher inflation, perhaps raising interest rates *temporarily* reduces inflation? What is the minimal simple economic model that produces this classic belief? (The qualifiers "simple" and "economic" are important.)

This section is rather long, because the important results are negative. It only takes one model to show a positive result. The interesting negative result here is that a suite of sensible modifications one might adduce to provide the desired sign do not work – sticky prices, monetary distortions and even backward-looking Phillips curves. One simply cannot say, for example, that sure, the Fisher relation means that raising interest rates raises inflation, but sticky prices overturn that result. They don't. A novel fiscal theory argument with long-term debt produces the desired negative sign, though rather deeply changes one's views of just what monetary policy is and how it works. The alternative possibility is that monetary policy necessarily relies on complex or non-economic ingredients, or that raising interest rates really does raise inflation, even in the short run.

6.1 Pure Fisher

The simplest starting point is a frictionless model incorporating a Fisher relation

$$i_t = r_t + E_t \pi_{t+1}$$

and the government debt valuation equation

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{17}$$

(One can flesh these out as the two important equilibrium conditions of a complete general equilibrium model with a constant endowment. See Cochrane (2005).) This model is "simple," "economic," and stable under an interest rate peg.

Figure 10 presents the impulse-response function for this case. In this model, raising the interest rate produces an immediate and complete rise in expected inflation.

The impulse-response function measures the path of $\{E_t \pi_{t+j}\}$ on the date t that the interest rate rise is announced – not necessarily the same as the day that interest rates actually rise. Thus, the Fisher equation alone allows in the impulse-response function an arbitrary one-period jump in inflation, upwards or downwards, coincident with the announcement of the policy change. The fact that it only ties down expected inflation has no impact on the impulse-response function – which plots the path of expected inflation – at other dates.

The top panel of Figure 10 shows some possible impulse-response functions when the interest rate rise is announced at the same time as it occurs. This is the usual assumption of VAR analysis. However, many real-world interest rate rises are announced long in advance, and we want to know the effect of such anticipated monetary policy as well. The bottom panel of Figure 10 shows possible impulse-response functions when the rise is announced three periods before it occurs.

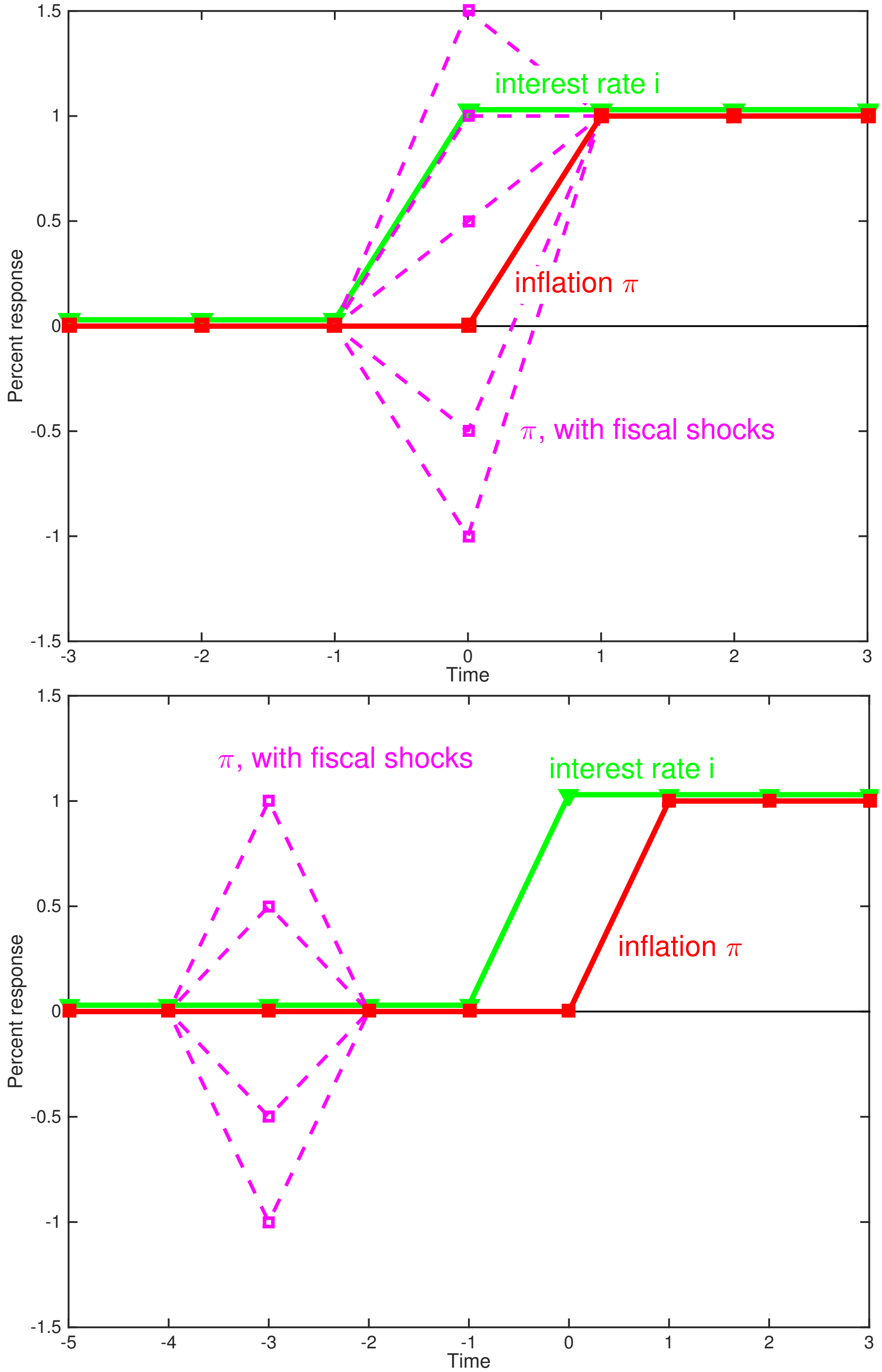


Figure 10: Response of inflation to a permanent interest rate increase. Frictionless model $i_t = r + E_t\pi_{t+1}$. Top: the rise is announced and implemented at time $t = 0$. Bottom: The rise is announced at $t = -3$. The main inflation line assumes one-period debt and no change in fiscal policy coincident with the monetary shock. The dashed "with fiscal shocks" lines add a change in primary surpluses coincident with the announcement of a higher interest rates.

Now, any unexpected inflation, which revalues government debt, must correspond to a revision in expectations of future surpluses. In (12), we took innovations of (17),

$$\frac{B_{t-1}}{P_{t-1}}(E_t - E_{t-1})\frac{P_{t-1}}{P_t} = (E_t - E_{t-1})\sum_{j=0}^{\infty}\beta^j s_{t+j}.$$

I think of "monetary policy" as a change in interest rates with no change in fiscal policy. This additional definition picks the central, $\delta = 0$ equilibrium as the effect of monetary policy, and it is to raise inflation throughout, with no difference between expected and unexpected policy.

How can we generate a temporary negative response? I start with the obvious economic ingredients – sticky prices and money. I then consider fiscal responses, to see if we can choose one of the negative δ paths.

7 Sticky prices

It is natural to hope that by adding sticky prices, the simple frictionless Fisherian model will display a temporary negative inflation response.

I use the standard optimizing sticky-price model,

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) \tag{18}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t \tag{19}$$

where x_t denotes the output gap, i_t is the nominal interest rate, and π_t is inflation.

The solution of this model for a given equilibrium interest rate path is derived in the Appendix. Inflation and output are two-sided geometrically-weighted distributed lags of the interest rate path,

$$\pi_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left[i_t + \sum_{j=1}^{\infty}\lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty}\lambda_2^{j} E_{t+1} i_{t+j}\right] + \sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j} \tag{20}$$

$$\kappa x_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left[\left(1-\beta\lambda_1^{-1}\right)\sum_{j=0}^{\infty}\lambda_1^{-j} i_{t-j} + \left(1-\beta\lambda_2^{-1}\right)\sum_{j=1}^{\infty}\lambda_2^{j} E_{t+1} i_{t+j}\right]$$
$$+ (1-\beta\lambda_1^{-1})\sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}, \tag{21}$$

where

$$\lambda_1 = \frac{(1+\beta+\kappa\sigma)+\sqrt{(1+\beta+\kappa\sigma)^2 - 4\beta}}{2} > 1 \tag{22}$$
$$\lambda_2 = \frac{(1+\beta+\kappa\sigma)-\sqrt{(1+\beta+\kappa\sigma)^2 - 4\beta}}{2} < 1 \tag{23}$$

Here, δ_{t+1}, with $E_t\delta_{t+1} = 0$, is an expectational shock indexing multiple equilibria. (In the frictionless and some simple sticky price models, $\delta_{t+1} = \pi_{t+1} - E_t\pi_{t+1}$. Here, since inflation π_{t+1} already responds to interest rate shocks at time $t+1$ in the $\delta_{t+1} = 0$ solution, δ_{t+1} represents an additional iid disturbance.)

7.1 Basic impulse-response function

Figure 11 presents the response of inflation and the output gap to a step function rise in the interest rate, using (20)-(21), and choosing the basic solution $\delta_0 = 0$. Throughout, unless otherwise noted, I use parameters

$$\beta = 0.97, \kappa = 0.2, \sigma = 1. \tag{24}$$

The solid lines of Figure 11 plot the responses to a pre-announced interest rate rise. The dashed line plots the response to an unexpected interest rate rise. Announced and surprise interest rate paths are the same after the announcement day. More generally, the response to this policy announced at any time before zero jumps up to match the anticipated-policy reaction on the day of announcement.

Inflation rises throughout the episode. Mathematically, that is a result of a two-sided moving average with positive weights in (20).

Output declines around the interest rate rise. When the nominal interest rate is

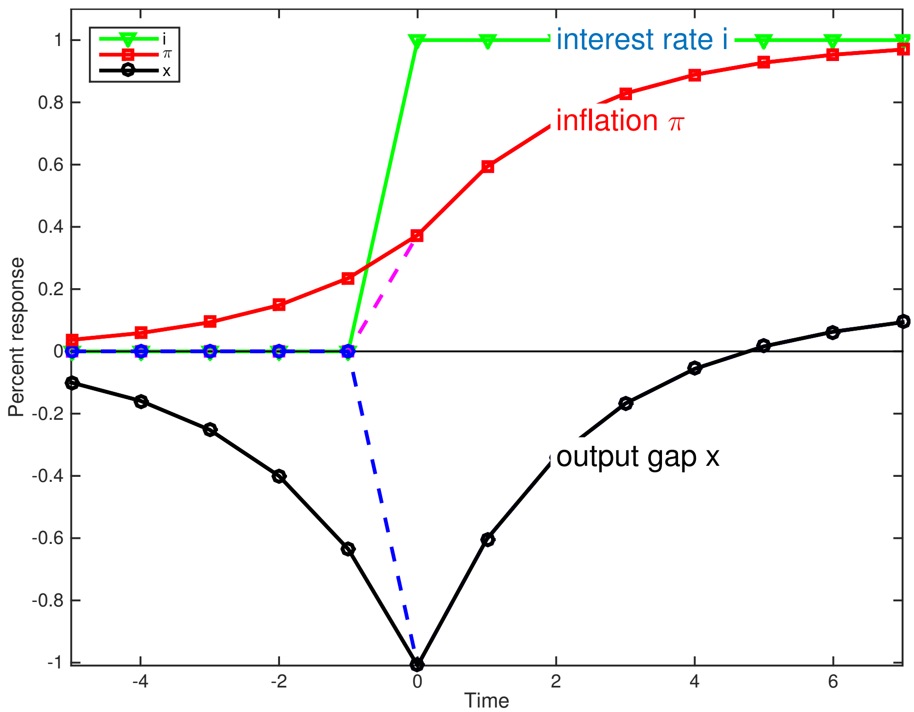


Figure 11: Response of inflation and output to a step function interest rate change in the standard IS - Phillips curve new-Keynesian model. The solid lines show the response to an expected change. The dashed lines show the response to an unexpected change. Paramters $\beta = 0.97$, $\kappa = 0.2$, $\sigma = 1$.

higher than the inflation rate, the real rate is high. Output is low when current and future real interest rates are high via intertemporal substitution. Equivalently, the forward looking Phillips curve (19) says that output is low when inflation is low relative to future inflation, i.e. when inflation is increasing.

Output eventually rises slightly, as the steady state of the Phillips curve (19) with $\beta < 1$ gives a slight increase in the level of output when inflation increases permanently. Using $\beta = 1$, there is no permanent output effect, and all graphs are otherwise visually indistinguishable. The positive inflation effect does not require a permanent output effect.

(Solving the Phillips curve (19) forward, one obtains inflation as a positive function of future output gaps. One might conjecture therefore that higher inflation requires positive output gaps in the long run. However, as β rises, and long-run output gaps decline, the $\beta^j E_t \pi_{t+j}$ term becomes more important. The solutions smoothly approach the case that long-run output gaps are zero, approaching from below, and expected inflation is one.)

In sum, this simple standard model gives a smoothed Fisherian inflation response to interest rate changes. One might have hoped that price stickiness would deliver the traditional view of a temporary decline in inflation. It does not.

The model does, however, generate the output decline that conventional intuition and most empirical work associates with monetary policy tightening. It therefore suggests a novel picture of monetary policy. Raising interest rates to cool off a booming economy, and lowering interest rates to stimulate a slow economy may make sense. Doing so just has a different effect on inflation than we might have thought. It paints a picture, not unlike recent experience and Fed statements such as Yellen (2016), in which monetary policy is primarily concerned with manipulating output, not inflation.

The sign is not affected, and magnitudes not greatly affected, by changes in the parameters. There isn't much you can do to an S shape. The parameters κ and σ enter together in the inflation response. Larger values speed up the dynamics, smoothly approaching the step function of the frictionless model as their product rises. Larger values of the parameter β slightly slow down the dynamics. Larger σ gives larger output effects with the same pattern.

Expected and unexpected policy have similar responses because the interest rate shock $i_t - E_{t-1} i_t$ does not appear as a separate right hand variable in the model's solutions (20)-(21), as it does in information-based Phillips curves such as Lucas (1972). As a result, in this class of models, expected monetary policy matters.

VARs often focus on the responses to unexpected policies, both out of historical tradition based on models such as Lucas' in which only unexpected monetary policy matters, and in order to try to identify the small exogenous movements in monetary policy. But our Fed telegraphs its intentions, often far in advance. So for policy and historical analysis, it is important to ask of models what is the effect of an expected policy change.

Output and inflation move ahead of the expected policy change. This fact reminds us that "forward guidance" matters, and that outcomes are affected by expectations, even when those expectations do not bear out.

7.2 Mean-reverting and stairstep rates

Empirical impulse-response functions usually find that the response of interest rates to an interest rate shock is mean-reverting, not a pure random walk as is the conceptual experiment of Figure 11. To think about that case, Figure 12 plots responses to an AR(1) interest rate shock.

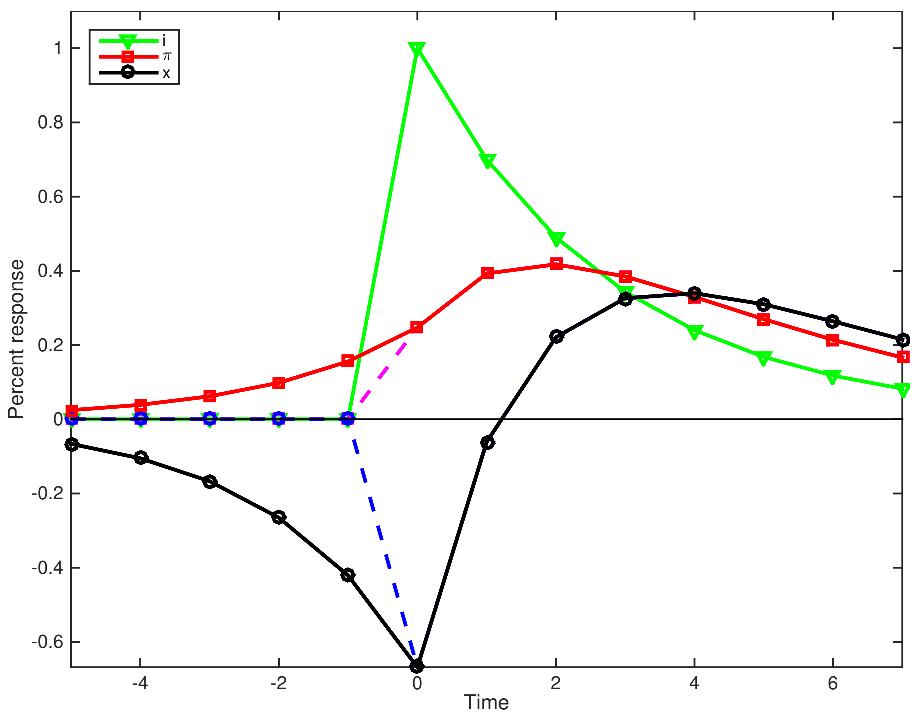


Figure 12: Response of inflation and output to a mean-reverting interest-rate path. Dashed lines are the response to an unexpected change. Solid lines are the response to an expected change.

One might have hoped that, since an expected rise in interest rates raises inflation, the expected declines in interest rates set off by the initial shock might have a contrary effect, depressing inflation or maybe even giving rise to a negative movement. Alas, that hope does not bear out. The responses in Figure 12 are similar to those of Figure

11 in the short run, with a long-run return to zero.

Figure 12 serves as an important reminder though: VARs that estimate transitory responses of interest rates to interest rates do not give us evidence on the long-run Fisher hypothesis. The zero bound experience tells us something that we could not observe in the transitory interest rate changes typical of the previous era.

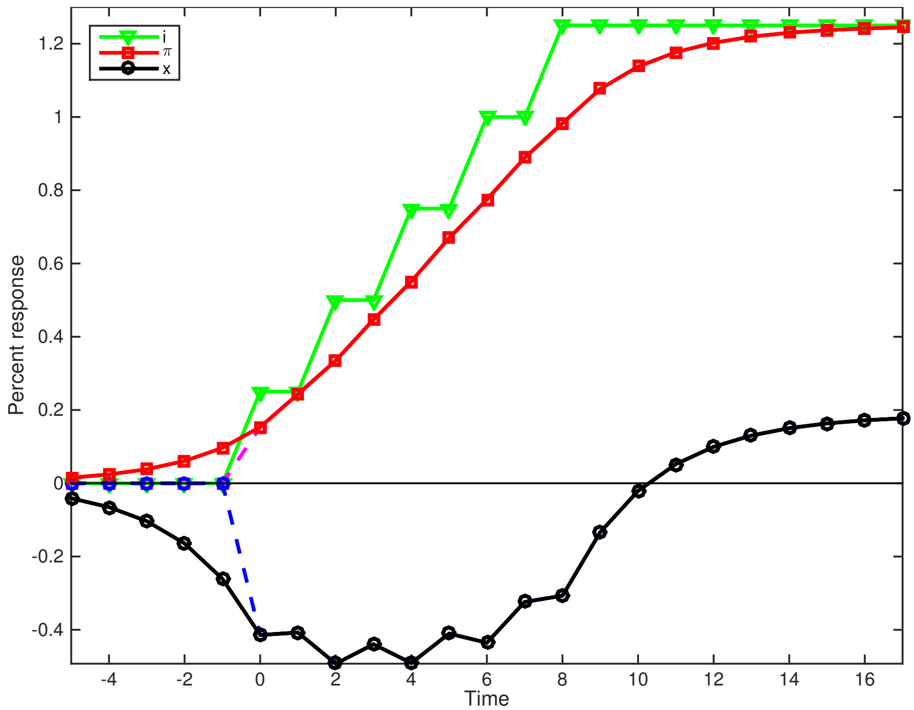


Figure 13: Response of inflation and output to a stairstep interest-rate path

Federal Reserve tightening typically takes the form of a well-anticipated steady set of stair step interest rate rises. Figure 13 presents the effects of such a policy. The result is qualitatively predictable from the other figures, though the smoothness of the inflation and output effects is noteworthy.

7.3 Taylor rules disclaimer

By solving for inflation and output given the equilibrium interest rate sequence $\{i_t\}$ I appear to assume that the Fed follows a time-varying peg without Taylor-rule responses. This is not the case.

The series $\{i_t\}$ represents a conjectured path for the *equilibrium* interest rate. Equations (20)-(21) tell us that *if* an equilibrium has an interest rate sequence $\{i_t\}$, then its inflation and output paths $\{\pi_t,\ x_t\}$ must follow (20)-(21). The "impulse response functions" are really the response of equilibrium inflation and equilibrium output to a policy change that also produces a step function rise in equilibrium interest rates. In $i_t = \phi\pi_t + v_t^i$, the *disturbance* $\{v_t^i\}$ is not the same thing as the *interest rate* $\{i_t\}$. In fact, with $\phi > 1$ they can even have a different sign as we will see shortly.

To match VARs, and to understand how inflation and output would respond if the Fed engineers a step function path of the nominal interest rate, this is exactly the question one wants to answer. Just how the Fed engineers the equilibrium interest rate path is not important, so long as it can do so.

Yes, the Fed can follow a time-varying or state-varying peg, and simply set the nominal interest rate path. But the Fed can also follow a Taylor rule with an active inflation response. For example, if the Fed follows a policy rule $i_t = \hat{i}_t + \phi(\pi_t - \pi_t^*) + v_t^i$, if the Fed changes monetary policy by shocks to $\{\hat{i}_t\}$, $\{\pi_t^*\}$, $\{v_t^i\}$, and if, on solving the the model, the equilibrium expected interest rates i_t follow the paths shown in Figures 11 - 13, then output and inflation also follow the paths shown in those figures. (Simply picking shocks π_t^* equal to the plotted inflation response will do.)

Werning (2012) innovated this clever idea of first finding equilibrium inflation and output given equilibrium interest rate paths, and then constructing the underlying Taylor rule. It simplifies the calculation, and makes it more general, as multiple underlying policies may give the same equilibrium interest rate path.

Below, I exhibit some underlying Taylor rule assumptions behind stairstep equilibrium interest rates. The important point here is that by calculating and plotting the response to a given interest rate path I do *not* make any assumptions about active vs. passive policy, and in particular I do not assume a time varying peg with $\phi = 0$. The calculations apply as much to active, $\phi > 1$ policy specifications as they do to passive specifications.

As previewed in Figure 10, there also remains the possibility of alternative equilibria, indexed by a one-time unexpected shocks δ on the date on which the new policy is announced, and each with a different fiscal consequence. Active Taylor rules and passive fiscal policy may act by selecting one of those equilibria, and then inducing the required change in fiscal surpluses. I return to this issue below also. Again, the quest is to see if *monetary* economics lowers inflation, not just because a monetary change induces a fiscal tightening. So, I first continue with the definition that "monetary policy" means changing interest rates and not changing or inducing changes in fiscal surpluses, and thus (to a good approximation, outlined below) picking the $\delta = 0$ equilibrium.

8 A fiscal-theoretic model with long term debt

Adding long-term debt produces a stable model in which a rise in interest rates can produce a temporary decline in inflation, with no change in fiscal surpluses. Sims (2011) makes this point in the context of a detailed continuous time model. Cochrane (2016b) shows how to solve Sims' model and boils it down to this central point.

Start with a frictionless model and hence use a constant real interest rate r and $\beta \equiv 1/(1+r)$ to discount surpluses. In the presence of long-term debt, the government debt valuation equation (11) becomes

$$\frac{\sum_{j=0}^{\infty} Q_t^{(j)} B_{t-1}^{(j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}, \tag{25}$$

where $B_{t-1}^{(j)}$ is the amount of zero-coupon debt that matures at time $t+j$ outstanding at the end of time $t-1$ and thus at the beginning of time t, and $Q_t^{(j)}$ is the time t nominal price of a j period discount bond.

When the Fed unexpectedly raises interest rates i_t, it lowers long-term bond prices $Q_t^{(j)}$. Debt $B_{t-1}^{(j)}$ is predetermined. By assumption, primary surpluses don't change. Hence, *the price level P_t must jump down by the same proportional amount as the decline in the nominal market value of the debt.* The sense that monetary policy works by affecting long-term bond prices is correct here, though the mechanism is utterly different from a Keynesian investment demand channel.

By raising nominal interest rates, the Fed still raises *expected* inflation uniformly, $i_t \approx r_t + E_t \pi_{t+1}$ still applies. However, the price level on the date of the announcement can jump downwards, as Figure 10 showed for innovations to fiscal surpluses coincident with the interest rate announcement, giving us the temporary decline in inflation that we're looking for.

This price level jump results in models such as this frictionless one, and the forward-looking new-Keynesian Phillips curve, in which the price level is a jump variable. In a model such as Sims (2011) with costs to swiftly changing prices, this jump can result in a smeared out period of disinflation. The jump in these simple models is a guide to the cumulative value of the disinflationary period.

The proportional change in market value of the debt, and hence the disinflation, is larger for interest rate changes that are expected to last longer, and so have a greater effect on long-term bond prices. The disinflation is also larger when there is more long-term debt around. Therefore, this channel predicts an interesting relationship between coefficients – the persistence of shock and the size of its effect – and an interesting state-dependence – monetary policy has larger effects when the maturity structure is longer.

Just how large a disinflation does this mechanism produce? Is it quantitatively significant, and hence a candidate to understand the apparent patterns in the data, or to guide policy?

Building up to the usual stair-step impulse-response function, suppose the interest rate $i = i_{t+j}$ is expected to last forever. The bond price is then $Q_t^{(j)} = 1/(1+i)^j$. Consider a geometric maturity structure, $B_{t-1}^{(j)} = \theta^j B_{t-1}$, so $\theta = 1$ is a perpetuity and $\theta = 0$ is one-period debt, and a constant surplus $s_t = s$. Now, the government debt valuation equation reads

$$\sum_{j=0}^{\infty} \frac{\theta^j}{(1+i)^j} \frac{B_{t-1}}{P_t} = \frac{1+i}{1+i-\theta} \frac{B_{t-1}}{P_t} = \frac{s}{1-\beta} \tag{26}$$

The continuous time analogue is prettier. With maturity $B_t^{(j)} = \vartheta e^{-\vartheta j}$

$$\vartheta \int_{j=0}^{\infty} e^{-ij} e^{-\vartheta j} dj \ \frac{B_t}{P_t} = \frac{\vartheta}{i+\vartheta} \frac{B_t}{P_t} = \frac{s}{r}. \tag{27}$$

Here $\vartheta = 0$ is the perpetuity and $\vartheta = \infty$ is instantaneous debt.

Now, suppose interest rates rise permanently and unexpectedly at time t. Denote by i^* the post-shock interest rate, and P_t^* the post-shock price level. Then, dividing (26) for the starred by the nonstarred case,

$$\frac{P_t^*}{P_t} = \frac{1+i-\theta}{1+i^*-\theta}\frac{1+i^*}{1+i}.$$

The prettier continuous time analogue is

$$\frac{P_t^*}{P_t} = \frac{i+\vartheta}{i^*+\vartheta}. \tag{28}$$

Now we can get a back of the envelope idea of the size of this effect and its crucial determinants. The longer the maturity, the stronger the effect. In the most extreme case, pairing this permanent interest rate rise with perpetual debt $\vartheta = 0$ – then the continuous-time formula gives $P_t^*/P_t = i/i^*$. A jump in interest rates from 2% to 3% causes the price level to drop to 2/3 of its previous value, a 33% decline!

However, the US doesn't issue that much long-term debt. Debt out to a 20 year maturity follows a geometric pattern with $\vartheta \approx 0.2$. In this case, a one percentage point interest rate rise implies $P_t^*/P_t = (0.2/0.21) = 0.95$, a 5% disinflation.

Shorter-lived interest rate rises, and announcements of future rate rises have less effect still. In the Appendix, I show that an interest rate rise from i to i^* that only lasts M years yields in place of (28),

$$\frac{P_t^*}{P} - 1 \approx \left(1 - e^{-\vartheta M}\right)\left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right) \tag{29}$$

For example, an interest rate rise that lasts 2 years $M = 2$ has only $1 - e^{-\vartheta 2} = 1 - e^{-0.2\times 2} \approx 1/3$ as large an effect, about 2% price level reduction.

An announcement of a future interest rate rise will lower long-term bond prices today, and cause the disinflation to get going before the actual interest rate rise. However, it has smaller effects because it only affects bonds of maturity longer than the announcement delay. An announcement that interest rates will rise in 3 years has no effect on 3 year bonds, and no effect on the price level if the maturity structure is limited to 3 year bonds. In this same simple geometric-maturity structure model, an announcement of an interest rate rise from i to i^* that starts in M years yields in place of (28),

$$\frac{P_t^*}{P_t} - 1 \approx e^{-\vartheta M}\left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right). \tag{30}$$

Thus, an interest rate rise that is announced two years ahead of time has a $e^{-(0.2\times 2)}$ or about 2/3 as much effect, or about 3% rather than 5% disinflation.

To give a more serious quantitative evaluation of this disinflationary effect, I use the 2014 zero-coupon U.S. Treasury debt outstanding provided by Hall and Sargent (2015) for $B_{t-1}^{(j)}$, and the 2014 zero coupon yield curve $\{Y^{(j)}\}$ from Gürkaynak, Sack, and Wright (2007) for bond prices $Q_t^{(j)} = \left(1/Y_t^{(j)}\right)^j$. I calculate the nominal market value of the debt as

$$\sum_{j=0}^{\infty} Q_t^{(j)} B_{t-1}^{(j)} = \sum_{j=0}^{\infty} \frac{1}{\left(Y_t^{(j)}\right)^j} B_{t-1}^{(j)}.$$

A permanent one percentage point increase in interest rates will increase yields at all maturities, so yields after the change are $Y^{*(j)} = \left(Y^{(j)} + 0.01\right)$. The downward price level jump is then given by

$$\frac{P_t^*}{P_t} = \frac{\sum_{j=0}^{\infty} \frac{1}{\left(Y^{(j)}\right)^j} B_{t-1}^{(j)}}{\sum_{j=0}^{\infty} \frac{1}{\left(Y^{(j)}+0.01\right)^j} B_{t-1}^{(j)}}.$$

To calculate the effects of a rate rise that lasts only M years, I first find the zero coupon forward curve before the change $F_t^{(j)} = Q_t^{(j)}/Q_t^{(j+1)}$. The effect of a M-year, one percentage point rise in interest rates is then

$$\frac{P_t^*}{P_t} = \frac{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{M-1}\frac{1}{F_t^{(j)}+0.01}\right)\left(\prod_{k=M}^{j-1}\frac{1}{F_t^{(j)}}\right) B_{t-1}^{(j)}}{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{F_t^{(j)}}\right) B_{t-1}^{(j)}}$$

Similarly, the effect of a permanent one percentage point interest rate rise, announced

M years in advance, is

$$\frac{P_t^*}{P_t} = \frac{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{M-1}\frac{1}{F_t^{(j)}}\right)\left(\prod_{k=M}^{j-1}\frac{1}{F_t^{(j)}+0.01}\right)B_{t-1}^{(j)}}{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{F_t^{(j)}}\right)B_{t-1}^{(j)}}$$

Figure 14 presents response functions, pairing these price level jumps with the frictionless Fisherian model $i_t = r + E_t\pi_{t+1}$. In the top panel, a surprise permanent 1 percentage point increase in interest rates drives a 5% disinflation, consistent with the back of the envelope calculation above. The long term debt response behaves exactly as a fiscal tightening of Figure 10, i.e. an announcement of 5% higher primary surpluses coincident with the monetary tightening. If the rate rise only lasts 3 years, in the the long end of the ballpark found in most VAR studies, long term bonds are less affected, so the disinflation is only about 2.5%.

The bottom panel of Figure 14 shows the response to an interest rate rise announced three years ahead of time. Though VARs look only at interest rate surprises, the bulk of actual interest rate changes are expected, so this response is most important to understand history, episodes, and policy. Like the fiscal shock of Figure 10, this disinflationary effect happens only on the announcement of the interest rate change, not on the day of the actual change. This is a vital point to remember when evaluating the plausibility of this mechanism relative to experience. The responses are smaller yet – 3% for the announced permanent rate increase, and 1.2% for the announced, three-year rate increase.

8.1 Long term debt and sticky prices

The same analysis applies to the sticky-price model, except that real interest variation changes the present value of surpluses. This consideration reduces further the size of the disinflationary effect. Allowing real interest rate variation and long-term debt, the valuation formula becomes

$$\sum_{j=0}^{\infty} Q_t^{(j)}\frac{B_{t-1}^{(j)}}{P_t} = E_t\sum_{j=0}^{\infty}\beta^j\frac{u'(c_{t+j})}{u'(c_t)}s_{t+j} = E_t\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{1+r_{t+k}}\right)s_{t+j}. \tag{31}$$

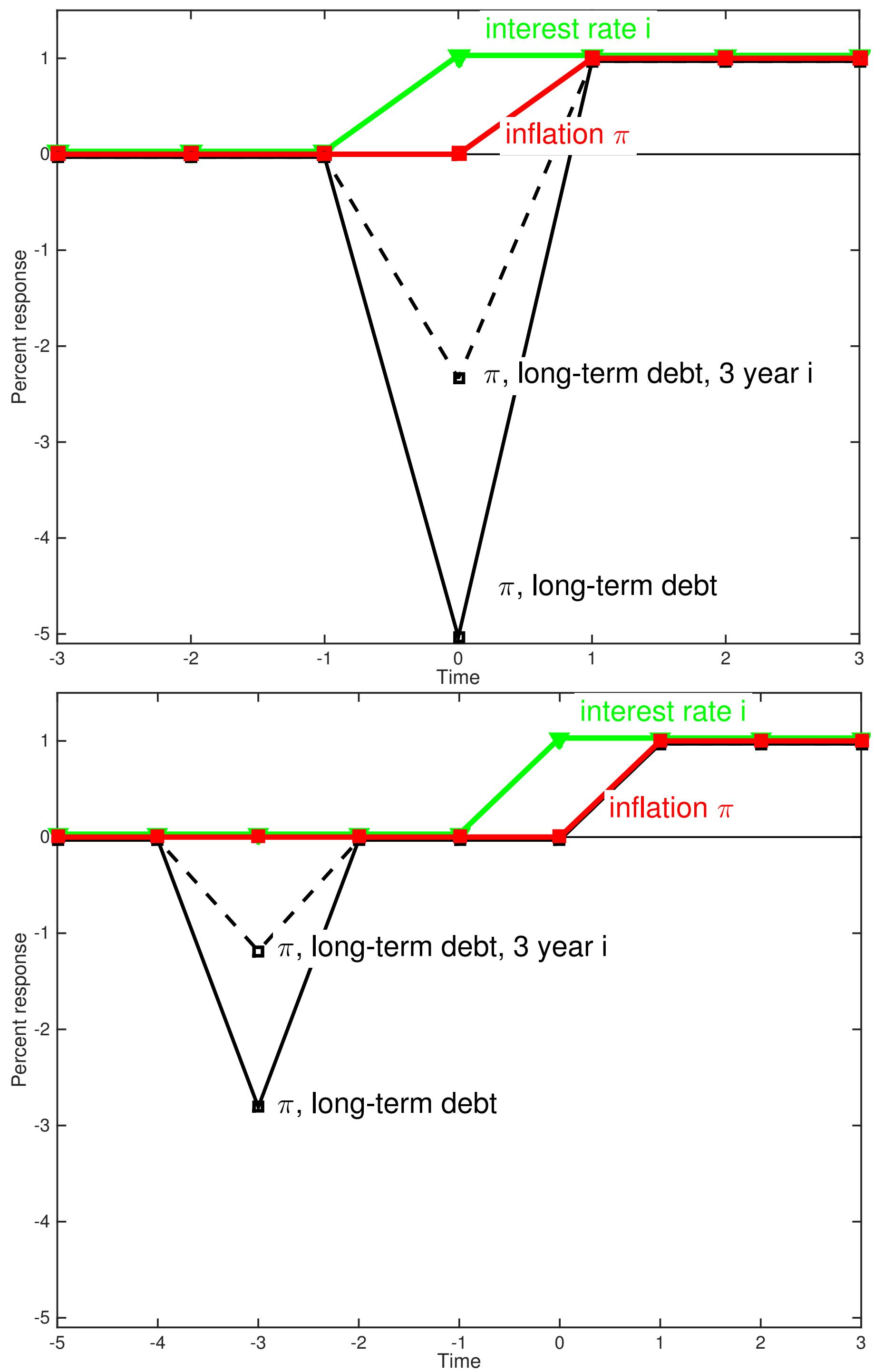


Figure 14: Response of inflation to interest rate changes with long-term debt. Top: response to interest rate change on the date of announcement. Bottom: response to a preannounced interest rate change. The responses are calibrated to the term structure of U.S. debt outstanding in 2014. Dashed lines give the response to a three-year rise in interest rates.

The first equality is the general formula; the second is an approximation reflecting the linearized nature of the new-Keynesian model we are working with, in which risk premiums do not vary over time.

We can always write nominal bond prices in terms of forward rates, and ignoring risk premiums in the term structure also in terms of expected nominal interest rates

$$Q_t^{(j)} = \prod_{k=0}^{j-1} \frac{1}{1+f_{t+k}} = E_t \prod_{k=0}^{j-1} \frac{1}{1+i_{t+k}}.$$

This expression helps us to compare left and right hand sides of the present value formula (31). The Fed still controls nominal interest rates, and hence bond prices at all maturities. Real rate variation and sticky prices make no difference to the left hand side side. Typically, when prices are sticky, higher nominal rates translate more into higher real rates and less into inflation. But such higher real rates reduce the real present value of surpluses on the right hand side of (31), to some extent matching the reduction in nominal present value of government debt on the left hand side, and thus requiring a lesser decline in price level.

For example, suppose inflation is perfectly sticky. Then, the rise in real rates on the right hand side of (31) exactly match the rise in nominal rates on the left, and there is no deflationary pressure at all.

To calculate the response function merging the standard new-Keynesian sticky price model with the fiscal theory and long-term debt, I again suppose interest rates start at their 2014 values, and I compute the market value of the debt. I then suppose nominal interest rates all rise by the interest rate response function, and I calculate the new nominal market value of the debt. I calculate the present value of an unchanged surplus using the government debt valuation formula (31). That consideration chooses a single value of P_t^*, equivalently of the multiple equilibria δ_t, on the announcement date. Equations are in the Appendix.

Figure 15 presents the response of inflation and output to an unexpected and permanent interest rate increase, shown as the top line. The solid line marked "Inflation π" and "$\Delta s = 0.00$" is the inflation path. In order to require no change in future surpluses, the devaluation of long-term debt now requires a 1.2% disinflation. Inflation is stable,

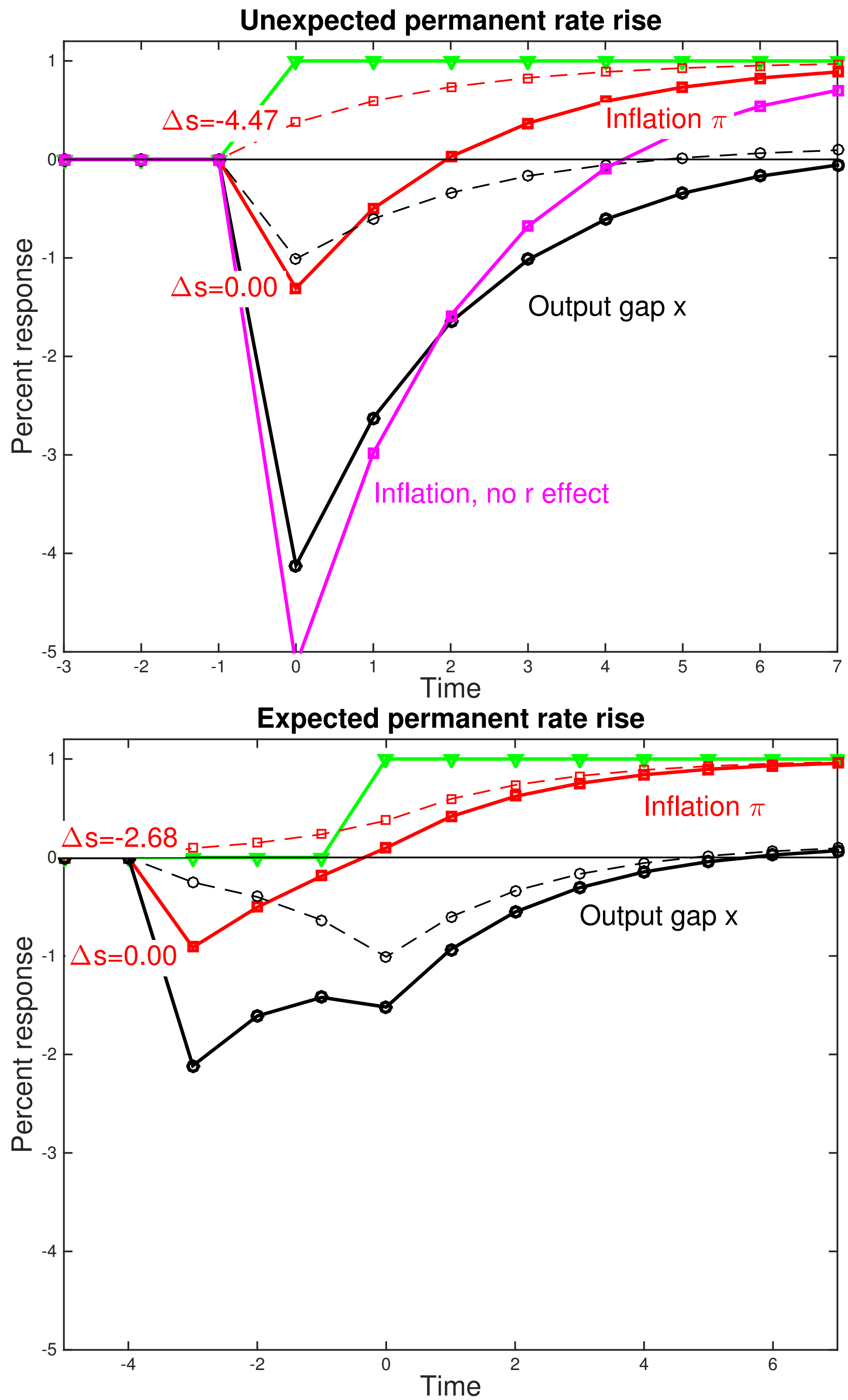
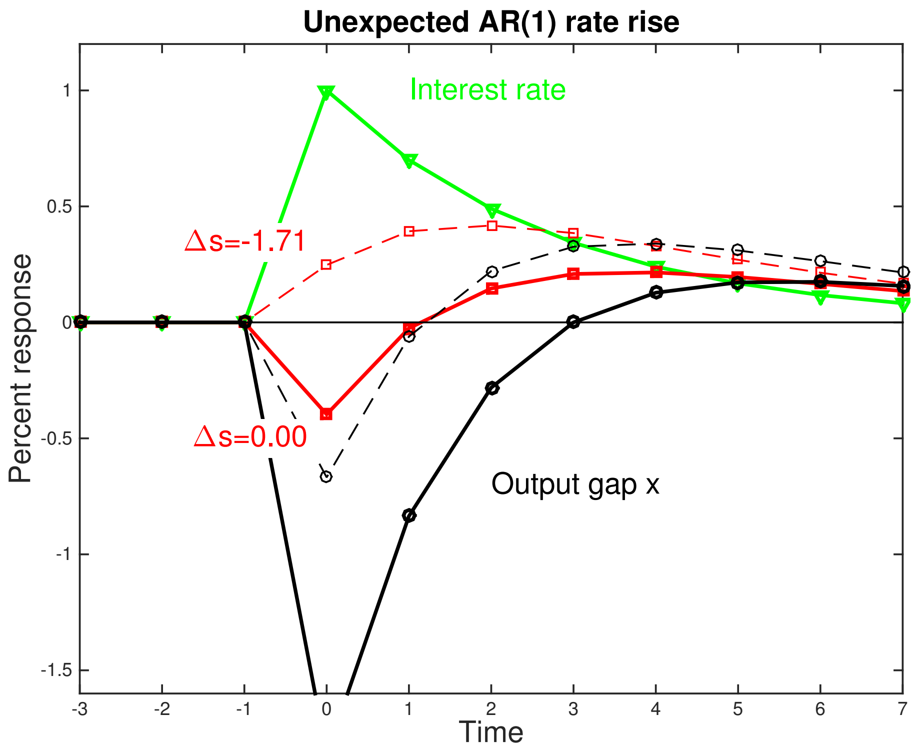


Figure 15: Response to permanent interest rate rises with long-term debt. I use the 2014 maturity structure of the debt to find the jump in price level that implies no change in primary surpluses. Thin dashed lines present the case with no additional shock, $\delta = 0$. The line "inflation, no r effect" in the first panel ignores the effect of rising real rates in devaluing future surpluses. "$\Delta s =$" gives the percent permanent change in primary surpluses associated with each inflation path.



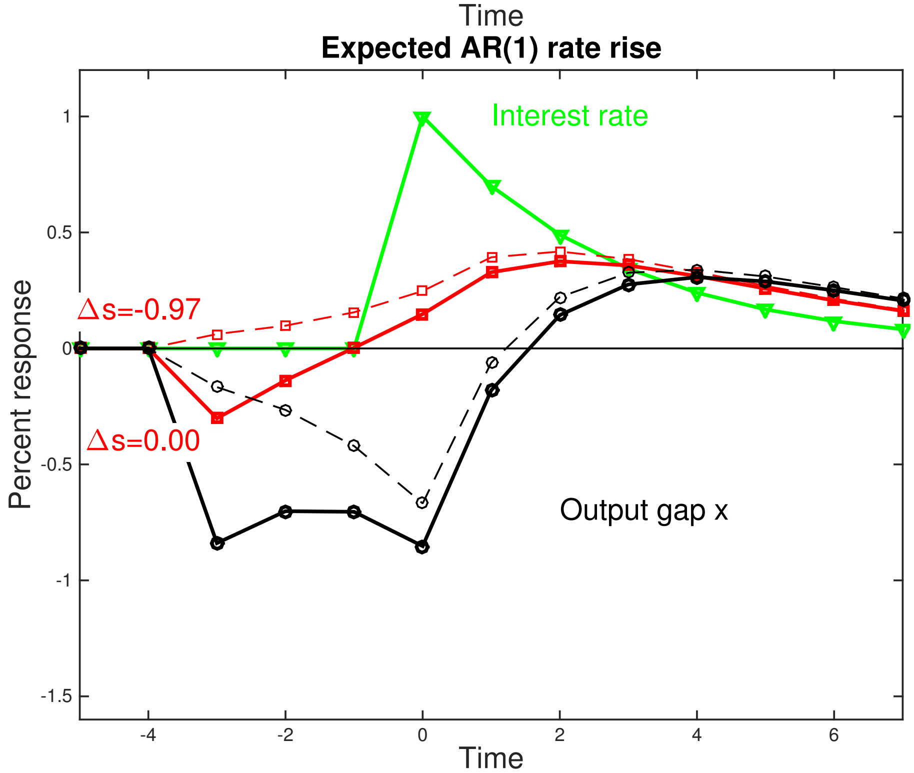


Figure 16: Response to transitory interest rate rises with long-term debt. I use the 2014 maturity structure of the debt to find the jump in price level that implies no change in primary surpluses. Thin dashed lines present the case with no additional shock, $\delta = 0$. "$\Delta s =$" gives the percent permanent change in primary surpluses associated with each inflation path. The interest rate reverts with a 0.7 AR(1) coefficient.

so eventually rises to meet the long-term interest rate. This is the most hopeful graph in this paper for an economically based model that gives the desired response function for monetary policy changes.

The upper thin dashed line in Figure 15 presents the previously calculated inflation path from Figure 11, with no extra shock $\delta_0 = 0$. I show below that this is nearly the same as the path with no change in primary surpluses in the presence of short-term debt. That line is marked "$\Delta s = -4.47$ in this case because, in the presence of long term debt which is devauled by higher interest rates, surpluses would have to decline by 4.47% permanently to boost inflation to this level.

The contrast between the dashed and solid inflation lines shows the effect of adding long-term debt to the model – and it is to give the long-sought period of disinflation.

The output gap lines in Figure 15 show that adding long-term debt, and thus producing a downward jump in the price level, increases the output effects of the interest-rate rise. With a forward-looking Phillips curve, the unexpected downward jump in the price level has no output effect at all. However, the expectation of a strong increase in inflation (from a lower level) drives output down.

The line marked "Inflation, no r effect" ignores the change in real interest rates on the right hand side of (31), to compare the pure devaluation of debt from higher nominal rates with the devaluation of surpluses from higher real rates. The higher real rates substantially lower the present value of surpluses, and make a big moderating difference to the initial disinflation. Models with this mechanism thus produce *less* disinflation from nominal interest rate rises when they have *more* sticky prices.

The bottom panel of Figure 15 shows the response when the interest rise is announced three years in advance. Again, these are models in which expected interest rate rises have effects, and most experience in the data and most policy interventions involve substantial expectations of future interest rate changes.

The higher interest rates now only affect bonds with three year or higher maturity. Thus, the downward inflation rate jump is smaller, only 1%. Output suffers a much less severe contraction, bottoming out at 2% not 4%.

As in the frictionless model, fiscal effects happen only on the day of announcement.

This is an important consideration in evaluating this channel. It will not rescue the old-Keynesian view that the interest rate rise itself sets off the disinflation. Nor does it rescue old-Keynesian instability.

The effects get uniformly smaller as the interest rate rise is expected further and further in the future. When the interest rate rise is expected after the maturity of the longest bond, the disinflationary effect vanishes entirely. Thus, this fiscal channel sensibly predicts smaller effects of expectations further in the future, and does not suffer from the forward-guidance puzzle.

Figure 16 presents the response to an unexpected (top) and expected (bottom) AR(1) rate rise. The unexpected transitory rate rise, though potentially unimportant for policy and for most historical events, is the slice of variation that is potentially recovered by VARs. (Though VARs don't attempt to orthogonalize monetary and fiscal policy shocks, $\Delta s = 0$ as I do here.) When comparing to Figure 15, note that Figure 16 has a larger vertical scale. The disinflation effect is now smaller still, less than 0.5% in both cases, down from 2% - 4%. The AR(1) interest rate rise has less effect on longer term bonds than a permanent rate rise. The expected AR(1) interest rate rise only really affects the value of bonds in the middle maturity range.

The big picture is that long-lived interest rate rises can produce disinflations on the same order of magnitude as the interest rate rises, and thus has the potential to explain the perceived effects of monetary policy.

8.2 Critical assumptions and intuition

To think about this channel more deeply, return to the constant real rate frictionless case,

$$\frac{\sum_{j=0}^{\infty} Q_t^{(j)} B_{t-1}^{(j)}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j}$$

Using the bond price, $Q_t^{(j)} = \beta^j E_t P_t / P_{t+j}$,

$$\sum_{j=0}^{\infty} \beta^j B_{t-1}^{(j)} E_t \left(\frac{1}{P_{t+j}} \right) = E_t \sum_{j=0}^{\infty} \beta^j s_{t+j} \tag{32}$$

When the Fed chooses higher nominal interest rates, and hence higher inflation and a higher future price level, it thereby devalues the long-dated coupons – $B_{t-1}^{(j)}$ are divided by larger P_{t+j}. This is great news for the Treasury – it doesn't have to raise as many real dollars s_{t+j} to pay off coupons. Rather than raise surpluses $s_{t+j} = B_{t-1}^{(j)}/P_t$, the Treasury could cut surpluses to $s_{t+j} = B_{t-1}^{(j)}/P_t^*$ and higher $P_t^* > P_t$ means less surpluses.

But in this exercise, the Treasury stubbornly refuses the gift: The Fed says, you can pay off the $1 coupons with (say) half as many real resources. But the Treasury says, no, we're going to insist on paying off the coupons with exactly the same real resources. Thus, the Treasury's refusal of the Fed's gift has the same effect as a fiscal contraction, a promise to raise future surpluses. Such a fiscal contraction causes government debt to be more valuable in real terms, and thus causes the price level to decline as people buy less goods and services to hold more government debt.

Having stated that intuition, you see how very important the fiscal reaction to monetary policy is in determining the outcome. Why does the Treasury stubbornly refuse to reduce surpluses when the Fed wants to inflate away long-dated coupons? Why does the Treasury not reduce future surpluses instead? If it is expected to do so, then we lose the disinflationary effect.

A second intuition is also valuable. Equation (32) acts like a "budget constraint" on the government. With no change in expected surpluses, any increase in expected price level at one date must be matched by a decrease in expected price level at another date. With long-term debt, expected surpluses control the moving average of current and expected future prices on the left side of (32). Within that moving average, the government can trade a higher price level at some days for a lower level of prices at other days, by varying the path of future debt sales, or equivalently by controlling nominal interest rates. Thus, when the government raises nominal interest rates, and thereby raises the price level at future dates relative to the current price level and devalues future coupons, it must also see a lower price level at date 0, and raise the value of near-term coupons.

As in the one-period debt case, (13), the Fed adjusts price levels at one day versus another by buying and selling debt with no changes in surpluses. To implement the interest rate target, and thus to raise, say, P_{t+j} while lowering P_{t+1}, the Fed will end up buying one-period debt and selling j-period debt. (Cochrane (2001) goes through the algebra.) Conversely, the Fed could lower say, P_{t+j} while raising P_{t+1}, by buying j-period debt and selling one-period debt, all with no change in surpluses.

Therefore, in this operation, monetary policy and quantitative easing (QE) operations operate in much the same way. Integrating QE and interest-rate policy in a single framework, requiring no market segmentation or other frictions, is attractive. It also sheds light on the puzzle why QE has had so little, or actually negative, effects on long-term inflation expectations.

This model, like any specific model, ties its explanation of a temporary decline in inflation to other predictions. Whether this is in fact the model of temporary disinflation that we seek depends on those other predictions as well. In this model, monetary policy only reduces inflation if it changes long term bond prices and the nominal market value of the debt.

This model does not easily produce a traditional stabilization. The decline in short-term inflation is inextricably linked to a rise in long-term inflation. If the Fed were to raise short-term rates, and people expected inflation to decline permanently, with the Fed eventually lowering short-term rates in the future – the traditional pattern of a monetary stabilization – then the value of nominal debt would rise, and this model would not produce the initial decline of inflation.

Sims (2011), elaborating on this model's mechanism, called it "stepping on a rake." He views the model as a description of the failed monetary stabilizations of the 1970s, in which interest rate increases produced temporary reductions of inflation that only came back more strongly later. To produce a successful inflation stabilization, a model of the 1980s, one needs something else. A natural candidate is to view the 1980s as a joint monetary-fiscal stabilization. The interest rate increases of the early 1980s were paired with fiscal reforms such as the 1986 tax act, and subsequent strong economic growth. Surpluses surged, which permanently reduce inflation. The natural policy for a permanent inflation reduction in this model remains to lower interest rates, along with some measure that really ensures fiscal expectations are unchanged; then wait out the temporary increase in inflation that results.

Furthermore, as I emphasized in the figures, this mechanism gives a disinflation when the interest rate rise is expected, not when it actually happens.

Finally, this mechanism for a short-run negative inflation effect does not immediately rationalize traditional policy. It is not necessarily wise or possible for the Fed to try to control inflation by exploiting this short-run negative sign. The negative sign only appears for *unexpected* policy changes, at the time of the news. Systematic policy, such as the $\phi\pi_t$ in $i_t = \phi\pi_t$, will not have any negative inflation effect. And getting the timing and dynamics just right are likely to be a challenge. Since the long-run effect is positive, there is a good case that to control inflation, the Fed should steady interest rates based on its long-run inflation goal and real-rate assessment, and not try to micromanage the path of inflation. with activist policy exploiting the transitory negative sign.

Most deeply, this model does not revive the instability of the old-Keynesian model behind both traditional activist policy advice and stabilizations. A transitory negative sign for unexpected interest rate rises, with a long-run positive, stable relationship between interest rates and inflation, is a very different animal from a transitory negative sign for all interest rate rises, with a long-run negative, unstable relationship. The latter does demand a Fed that actively exploits the negative sign. The former does not.

If we accept this direction, two important next steps follow. First, changes in interest rates with fixed surpluses are a useful textbook, problem-set sort of assumption. They are worth working out to understand mechanisms. But fixed or “exogenous” surpluses are not necessary for the theory. And fixed or “exogenous” surpluses are a terrible assumption for policy, econometric or historical analysis. Just how will the Treasury respond to inflation? Surely not zero, as I specified here. Yet it is the crucial assumption to understand how interest rates affect inflation. More deeply, both fiscal (surpluses) and monetary (interest rates) policy react to the same underlying sets of events. Any historical episode represents a set of monetary *and* fiscal responses to other events. A movement in interest rates with no change in expected surpluses just doesn’t happen. So the first step is to think much harder about the path of surpluses, how surpluses react to economic events, and how fiscal policy reacts to the same underlying economic shocks that motivate the change in interest rates.

Second, of course, one must move beyond the extremely simple model presented here to more detailed models capable of matching dynamics. Sims (2011) is a good example, adding a preference for smooth consumption, a monetary policy rule with output and price reactions and inertia, and a fiscal policy rule that raises surpluses in good times. He produces a hump-shaped inflation response curve in place of my downward jump followed by rise.

In sum, this mechanism produces a model in which inflation is stable, but has the desired short-run negative inflation response. The response is quantitatively important and could potentially describe empirical work. It describes interest rate policy and quantitative easing in one breath. However, comes with some unusual additional predictions – it does not give the conventional description of a long-run stabilization by monetary policy alone, does not describe any negative effect of expected rate changes, and does not offer an easy justification that the Fed should exploit the negative sign in regular policy making. Most of all, it relies on a fundamentally unconventional mechanism. Though flight from or to government debt feels like "aggregate demand" to those living in such an economy, the mechanism is entirely fiscal-theoretic – it acts exactly as a change in expected surpluses with one-period debt acts. Completely absent from the story are conventional mechanisms such as IS curves, the effect of interest rates on investment "demand," monetary distortions and even pricing frictions.

Now that we see in detail the car we are driving, before taking this fork in the road, let us make sure that standard monetary economics cannot be saved.

9 Money

Perhaps monetary distortions, in addition to pricing distortions, will give us the traditional result. Perhaps when interest rate increases were accomplished by reducing the supply of non-interest-bearing reserves, that reduction in money produced a temporary decline in inflation that simply raising the interest rate on excess reserves will not produce. Such a finding would, however, suggest that raising interest rates by simply raising the rate paid on abundant excess reserves will not have the same temporary disinflationary effect as past history suggests.

I introduce money in the utility function, nonseparable from consumption, so that changes in money, induced by interest rate changes, affect the marginal utility of consumption, and thus the intertemporal-substitution equation.

Woodford (2003) (p. 111) begins an analysis of this specification. But Woodford quickly abandons money to produce a theory that is independent of monetary frictions, and does not work out the effects of monetary policy with money. If theory following that choice now does not produce the desired outcome, perhaps we should revisit the decision to drop money from the analysis.

The detailed presentation is in the Appendix. The bottom line is a generalization of the intertemporal-substitution condition (18), to:

$$x_t = E_t x_{t+1} + (\sigma - \xi)\left(\frac{m}{c}\right) E_t\left[\left(i_{t+1} - i^m_{t+1}\right) - \left(i_t - i^m_t\right)\right] - \sigma\left(i_t - E_t \pi_{t+1}\right). \tag{33}$$

The presence of money in the utility function has no effect on firm pricing decisions and hence on the Phillips curve (19). Here, ξ is the interest-elasticity of money demand. Evidence such as Figure 8 and literature surveyed in the Appendix suggests $d\log(m)/d\log(i) = -\xi \approx 0.1$. The value m/c is the steady state ratio of real money holdings to consumption. The larger this value, the more important monetary distortions. The quantity i^m_t is the interest rate paid on money.

Equation (33) differs from its standard counterpart (18) by the middle, change-in-interest rate term. Equation (33) reverts to (18) if utility is separable between money and consumption $(\sigma - \xi) = 0$, if m/c goes to zero, or if money pays the same interest rate as bonds $i = i^m$.

The expression $m/c\ (i_t - i^m_t)$ represents the proportional interest costs of holding money. The middle term following $(\sigma - \xi)$ represents the expected change in those proportional interest costs. An expected increase in interest costs of holding money, induces the consumer to shift consumption from the future, when holding the money needed to purchase consumption goods will be relatively expensive, towards the present. It acts just like a lower real interest rate to induce an intertemporal reallocation of consumption.

The presence of expected changes in interest rates brings to the model a mechanism that one can detect in verbal commentary: the sense that *changes* in interest rates affect the economy as well as the level of interest rates.

However, monetary distortions only matter in this model if there is an *expected change in future* interest rate differentials. Expected, change, and future are all crucial modifiers. A higher or lower steady state level of the interest cost of holding money does not raise or depress today's consumption relative to future consumption. An unexpected change in interest costs has no monetary effect at all, since $E_t\left(i_{t+1}-i_t\right)=0$ throughout.

The model solution is essentially unchanged. The extra term in the intertemporal substitution equation (33) amounts to a slightly more complex forcing process involving expected changes in interest rates as well as the level of interest rates. One simply replaces i_t in (20)-(21) with z_t defined by

$$z_t \equiv i_t - \left(\frac{\sigma-\xi}{\sigma}\right)\left(\frac{m}{c}\right) E_t\left[\left(i_{t+1}-i_{t+1}^m\right)-\left(i_t-i_t^m\right)\right].$$

The slight subtlety is that this forcing process is the change in *expected* interest differentials. Lag operators must apply to the E_t as well as what's inside. Inflation depends on past expectations of interest rate changes, not to past interest rate changes themselves.

9.1 Impulse-response functions

I start with the traditional specification that the interest on money $i_t^m=0$, so that increases in the nominal interest rate are synonymous with monetary distortions. Figure 17 plots the response function to our expected and unexpected interest rate step with money distortions $m/c=0,\ 2,\ 4$.

For the unexpected interest rate rise, shown in dashed lines, the presence of money makes no difference at all. The dashed lines are the same for all values of m/c, and all the same as previously, and the model remains stubbornly Fisherian. This is an important negative result. Money can only affect the response to *expected* interest rate changes.

The response to an expected interest rate rise, shown in solid lines, is affected by the monetary distortion. As we increase the size of the monetary distortion m/c, inflation is lower in the short run. For $m/c=4$, we get the classic shape of the impulse response function. The announced interest rate rise produces a temporary decline in inflation,

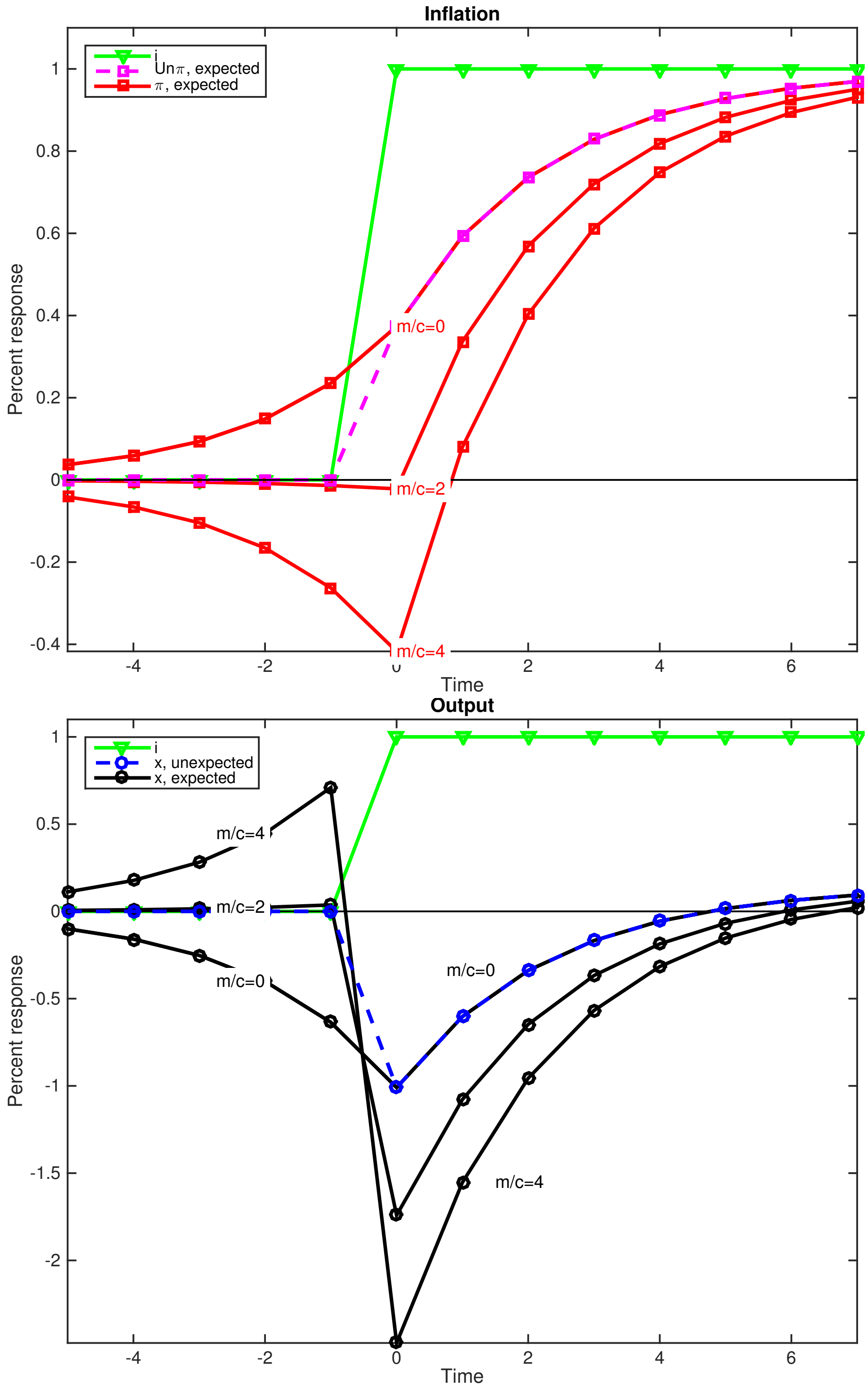


Figure 17: Response of inflation and output to an interest rate rise; model with money. The three cases are $m/c = 0,\ 2,\ 4$. Solid lines are an expected interest rate rise, dashed lines are an unexpected rise.

and then eventually the Fisher effect takes over and inflation increases.

The only time-difference in interest costs comes at time 0. Larger m/c induces the consumer to shift consumption to times before 0, to consume when the interest costs of holding the necessary money are lower. Output is high when inflation is decreasing, and vice versa, so this pattern of output corresponds to lower inflation before time 0 and higher inflation afterward.

The $m/c = 4$ curve seems like a great success, until one ponders the size of the monetary distortion – non-interest bearing money holdings equal, on average, to four years of output. This model is not carefully calibrated, but $m/c = 4$ is still an order of magnitude or more too large.

Equation (33) suggests that raising σ, which multiplies m/c, may substitute for a large m/c, by magnifying the effect on consumption of a given monetary distortion. Now, higher σ also magnifies the last term, which induces Fisherian dynamics. But in our response functions, the middle term multiplies a one-time shock, where the last term multiplies the entire higher step. Thus, raising σ can raise the relative importance of the one-time shock in the dynamics of inflation.

Figure 18 investigates the effect of changing the intertemporal substitution elasticity σ. Since an unexpected interest rate rise again has no monetary effect, I present only the case of an expected interest rate rise.

The left two panels, labeled $mc = 0$ in Figure 18, show the effect of varying σ in the model without money. In this model, σ and κ enter symmetrically in the determination of inflation. Raising σ increases the speed of the dynamics, pulling the S shaped response closer to the step that holds in a frictionless model. Raising the speed of the dynamics lowers inflation in the early period, a step in the direction of the conventional belief. But raising σ without money can never produce a negative effect on inflation.

The right two panels of Figure 18 with $m/c > 0$ show how increasing σ can work together with a monetary friction. At $m/c = 1$, increasing σ from $\sigma = 1$ to $\sigma = 3$ produces a slight decline in inflation before the inevitable rise. The subsequent rise is quicker; the main effect here has been to borrow inflation from the future. To get a substantial negative effect, one must increase either σ or m/c even more. The line

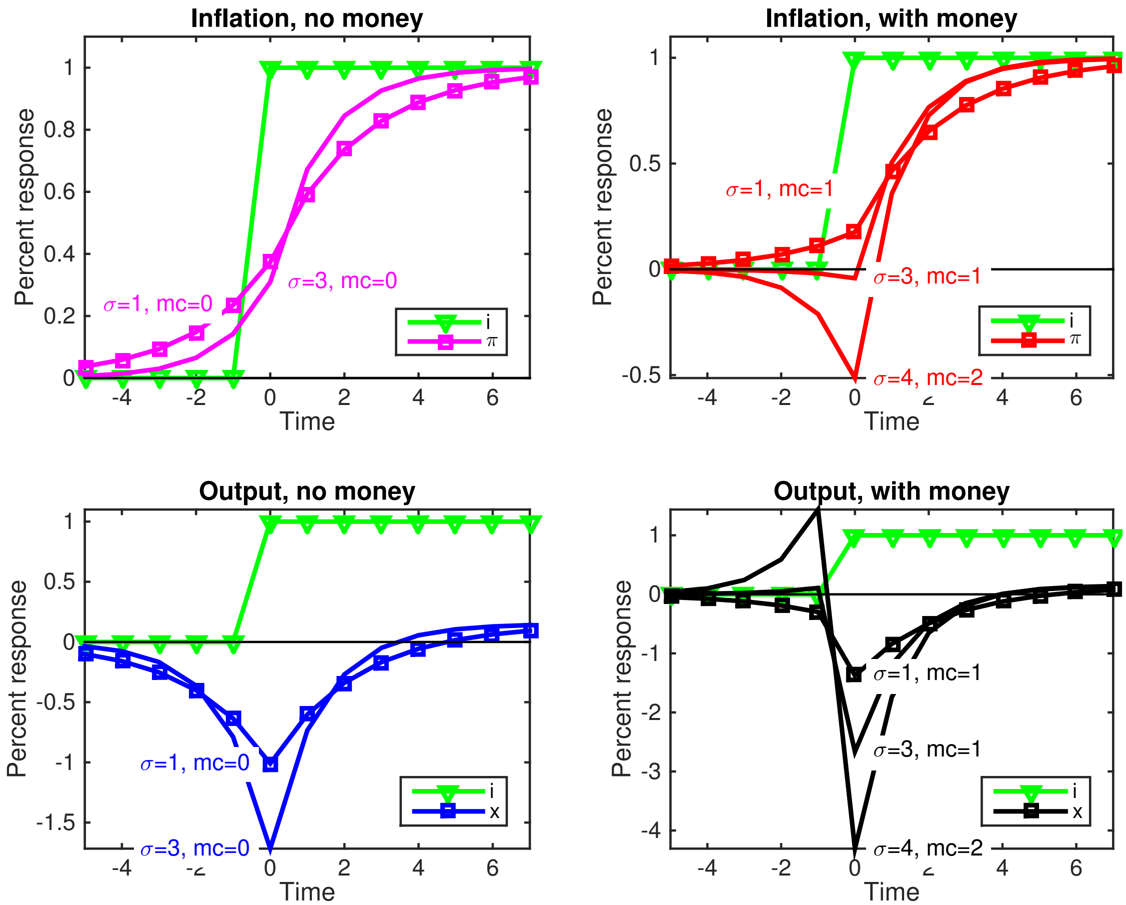


Figure 18: Response of output and inflation to an expected interest rate step; model with money and varying intertemporal substitution elasticity σ.

$\sigma = 4,\ m/c = 2$ produces about the same inflation decline as $\sigma = 1, m/c = 4$ produced in Figure 17.

So, higher σ can help to produce a temporary dip in inflation, largely by speeding up dynamics. Alas, $\sigma = 1$ was already above most estimates and calibrations. A coefficient $\sigma = 3$ implies that a one percentage point increase in the real interest rate induces a three percentage point increase in consumption growth, which is well beyond most estimates. And $m/c = 1$ is already at least twice as big as one can reasonably defend.

In sum, these calculations show what it takes to produce the standard view: For an anticipated interest rate rise only, money in the model can induce lower inflation than a model without monetary frictions produces. If we either have very large money holdings subject to the distortion, or a very large intertemporal substitution elasticity, the effect can be large enough to produce a short-run decline in inflation. Adding money to the model in this way has absolutely no effect on responses to an unexpected permanent interest rate rise.

9.2 Money and transitory rate shocks

Since expected changes in interest rates are the crucial mechanism in this model, perhaps putting in more reasonable interest rate dynamics can revive the desired inflation dynamics.

Figure 19 shows the response function to an AR(1) interest rates shock. Money does affect the response functions. And, that effect is uniformly to *raise* inflation. The expected decline in interest costs posed by the AR(1) reversion after the shock shifts consumption from the present to the future, and inflation rises when output is low.

With expected interest rate dynamics, the unanticipated rate rise now has monetary effects, shown in the dashed lines. These too are uniformly in the direction of higher inflation as monetary frictions increase.

9.3 Interest spread policy

The Federal Reserve is contemplating varying the interest it pays on reserves as separate policy tool. By changing the interest on reserves, the Fed can affect money demand without changing the nominal rate. Thus, it can focus on the monetary effects on demand without the direct intertemporal substitution effects.

Figure 20 presents a calculation. Here, the Fed raises the interest on reserves i^m by one percentage point, with no change in the nominal interest rate i.

One can debate whether we should call such a rise in interest on reserves “expansionary” or “contractionary” policy a priori. Raising interest on reserves is often considered contractionary, as it encourages banks to sit on reserves rather than to pursue lending. On the other hand, raising interest on reserves lowers the spread between reserves and other instruments, and so encourages the accumulation of money, which

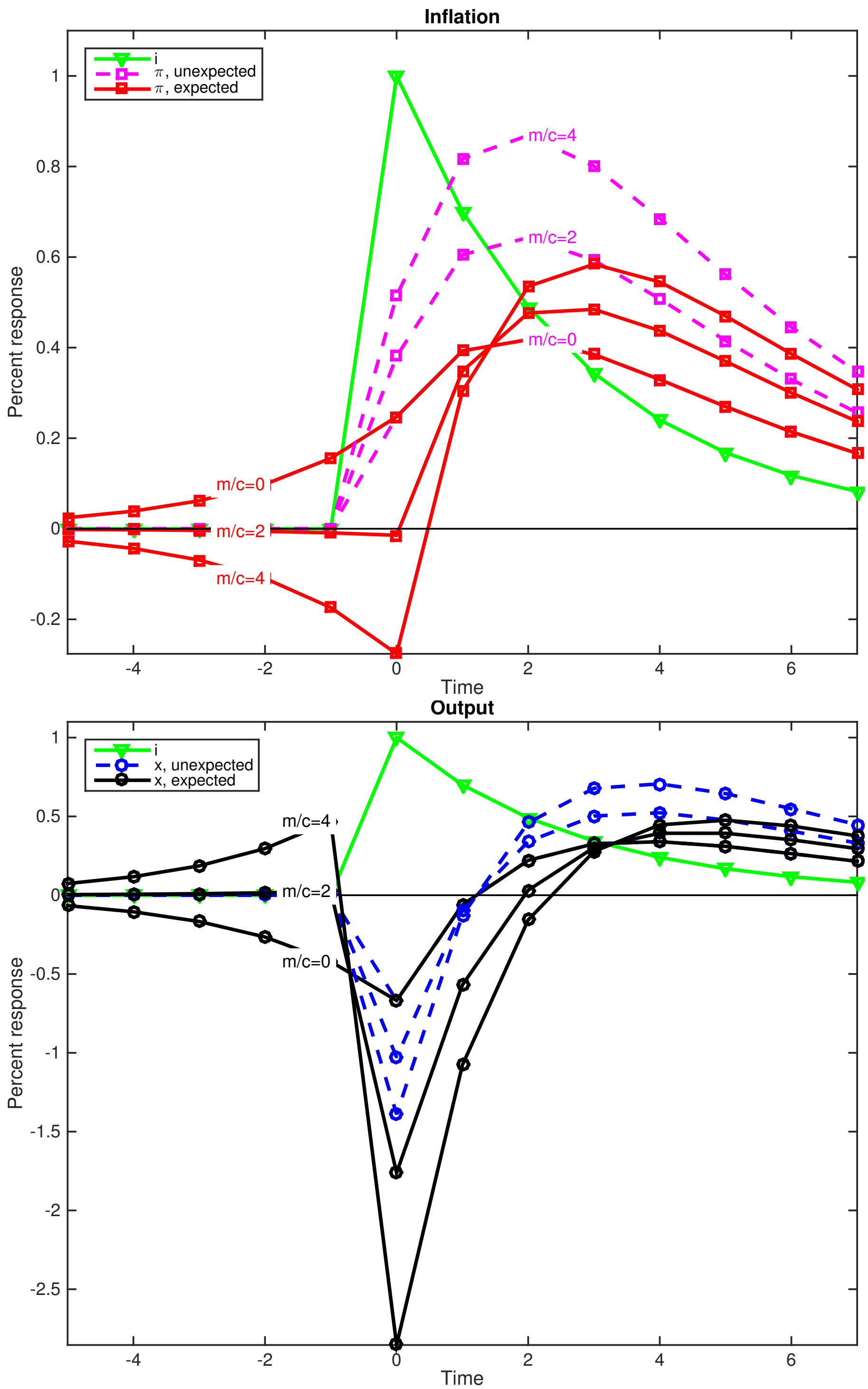


Figure 19: Response of inflation and output to a temporary rate rise, model with money. Dashed lines are the response to an unexpected rise, solid lines are the response to an expected rise.

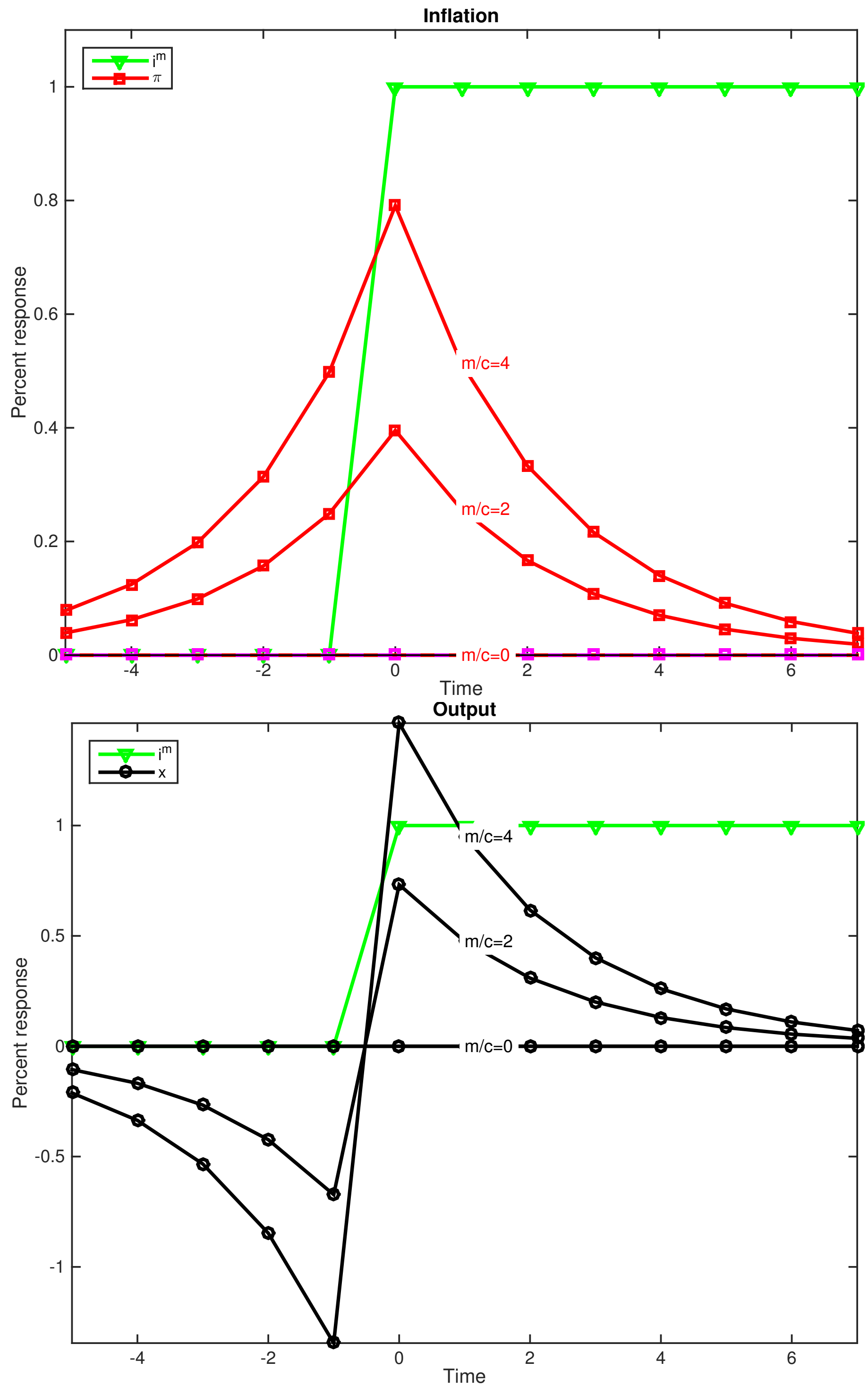


Figure 20: Response to a permanent rise in the rate paid on reserves, holding the nominal interest rate constant.

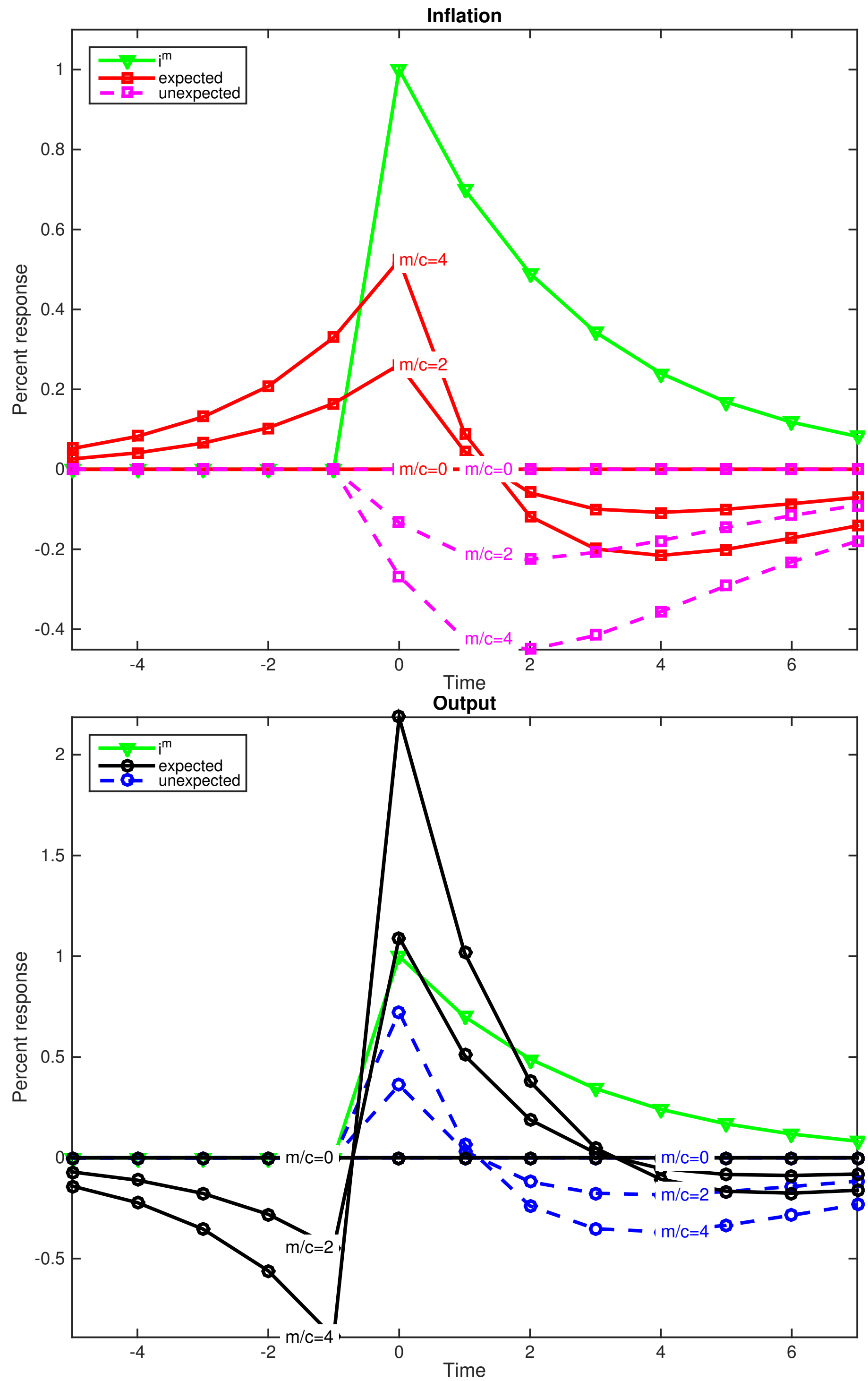


Figure 21: Response to a transitory rise in the rate paid on reserves, holding the nominal interest rate constant. Solid lines are the response to an expected change; dashed lines are the response to an unexpected change.

one might consider to be expansionary.

Again, the response to an unexpected rise in the interest on reserves is exactly zero. The intertemporal substitution mechanism only operates when the expected future is different from the present.

An expected rise in interest on reserves raises inflation throughout. Output declines ahead of the change, and rises after the change. Money is cheaper to hold after the rise, encouraging consumers to postpone consumption. With a forward looking Phillips curve, lower output corresponds to rising inflation, and vice versa.

Figure 21 graphs the effects of a transitory increase in the interest on reserves, which is expected to die out with an AR(1) pattern, again holding the nominal rate unchanged.

In this case, the unexpected change (dashed lines) lowers inflation. The shock itself has no effect, because it was unexpected. However once the shock as passed, consumers expect interest on reserves to decline, and expect interest costs of holding money to rise. This change induces them to bring consumption forward to the periods just after $t = 0$, as shown in the bottom panel. Higher output means declining inflation to the forward-looking Phillips curve as shown in the upper panel.

One might celebrate the first unambiguous negative inflationary effect of a tightening, but the mechanism is far from traditional. The rise in interest on reserves, being unexpected, has no effect at all. The rise allows for an expected decline in interest on reserves, and it is this expected decline which does all the work. An expected decline with no initial rise would have the same effect. And the actual interest rate is constant throughout.

The expected case of Figure 21 adds the effects of the anticipated rise in interest on reserves to this story. The expected rise induces consumers to postpone consumption to the period just after the rise, doubly increasing output then, but also driving up inflation.

9.4 Money summary

The basic mechanism for "demand" in this, as in all new-Keynesian models, is intertemporal substitution, changes in the margin of current versus future consumption. Adding money to the model distorts the tradeoff of current vs. future consumption. In making that tradeoff, the consumer thinks about the actual real interest rate, but also the relative costs of holding money necessary to make purchases today vs. that cost in the future. A higher interest cost of holding money in the future is, like a higher real interest rate, an inducement to consume now.

Alas, this mechanism is quantitatively small. Relative to actual changes in real interest rates, the distortions to intertemporal incentives from greater or lesser costs of holding money are second-order. We just don't hold that much non-interest-bearing money.

Also, this mechanism does not give rise to classic intuition. Interest costs of money holdings only affect "demand" if people *expect* higher or lower interest costs in the future than they experience today. The level of interest costs has no effect.

10 The Phillips curve

Empirically, lags seem important in Phillips curves. The forward-looking Phillips curve (19) specifies that output is higher when inflation is high relative to future inflation, i.e. when inflation is declining. Though all Phillips curves fit the data poorly, especially recently, output is better related to high inflation relative to past inflation, i.e. when inflation is rising (Mankiw and Reis (2002)).

Theoretically, the pure forward looking Phillips curve is not central. Though it does some violence to the "economic" criterion for the simple baseline theory that we are searching for, we should check if the short or long-run neo-Fisherian conclusions can be escaped by adding past inflation to the Phillips curve.

The simplest approach is to consider a static Phillips curve. This specification is the $\beta \to 0$ limit of the three equation model (18)-(19). Kocherlakota (2016) provides detailed micro-foundations for a static Phillips curve.

So consider

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \tag{34}$$

$$\pi_t = \kappa x_t. \tag{35}$$

The equilibrium is simply

$$E_t \pi_{t+1} = \frac{1}{1+\sigma\kappa}\pi_t + \frac{\sigma\kappa}{1+\sigma\kappa} i_t \tag{36}$$

and hence

$$\pi_t = \sigma\kappa \sum_{j=1}^{\infty} \frac{1}{(1+\sigma\kappa)^j} i_{t-j} + \sum_{j=0}^{\infty} \frac{1}{(1+\sigma\kappa)^j}\delta_{t-j}. \tag{37}$$

The dynamics are stable, and inflation responds positively to interest rates throughout. In fact, they are *exactly* the same dynamics as we found in section 3 for a static IS curve and fully forward-looking Phillips curve. Repeating the equations for clarity,

$$x_t = -\sigma(i_t - E_t \pi_{t+1}) \tag{38}$$

$$\pi_t = E_t \pi_{t+1} + \kappa x_t \tag{39}$$

leads exactly to the same conditions (36)-(37).

Thus, Figure 6 already plots the response function for the static Phillips curve case (34)-(35) - and inflation rises smoothly throughout.

Even eliminating both intertemporal terms,

$$x_t = -\sigma(i_t - E_t \pi_{t+1})$$

$$\pi_t = \kappa x_t$$

produces

$$E_t \pi_{t+1} = \frac{1}{\sigma\kappa}\pi_t + i_t.$$

The model is stable or unstable depending on $\sigma\kappa$, but the sign on interest rates remains positive.

A backwards-looking accelerationist Phillips curve is more realistic. This model also adds some reassurance that the Fisherian results do not result from a permanent inflation-output tradeoff, present both in the static Philips curve and the new-Keynesian model with $\beta < 1$. Consider

$$\pi_t = \pi^e_{t-1} + \kappa x_t$$

$$\pi^e_t = (1-\lambda)\sum_{j=0}^{\infty} \lambda^j \pi_{t-j}.$$

Substituting the output gap from the usual intertemporal IS curve (34) ,

$$(\pi_t - \pi^e_{t-1}) = (E_t\pi_{t+1} - \pi^e_t) - \sigma\kappa(i_t - E_t\pi_{t+1})$$

$$(1+\sigma\kappa)E_t\pi_{t+1} = \pi_t + \pi^e_t - \pi^e_{t-1} + \sigma\kappa i_t$$

$$(1+\sigma\kappa)E_t\pi_{t+1} = \pi_t + (1-\lambda)\left[\sum_{j=0}^{\infty}\lambda^j \Delta\pi_{t-j}\right] + \sigma\kappa i_t$$

Figure 22 plots the impulse-response function for this model. It is Fisherian throughout. Now the tightening *increases* the output gap. In some sense by giving the "right" (old-Keynesian) sign of the relationship between output and inflation, since a rise in interest rates gives the "wrong" (positive) effect on inflation, it also gives the "wrong" (positive) effect on output.

This response function is the same for expected as for unexpected monetary policy, which at least should give some comfort to the traditional view that the effects of policy have to await some action by the Federal Reserve.

Figure 22 plots the $\delta = 0$ equilibrium. One can add a negative jump with fiscal tightening consequence to deliver a negative inflation response on the announcement day, but that is a different mechanism which I study in detail elsewhere.

The Appendix considers a model with both forward and lagged terms in the Phillips curve.

$$\pi_t = \kappa\left(x_t + E_t\sum_{j=1}^{\infty}\phi^j x_{t+j} + \sum_{j=1}^{\infty}\rho^j x_{t-j}\right).$$

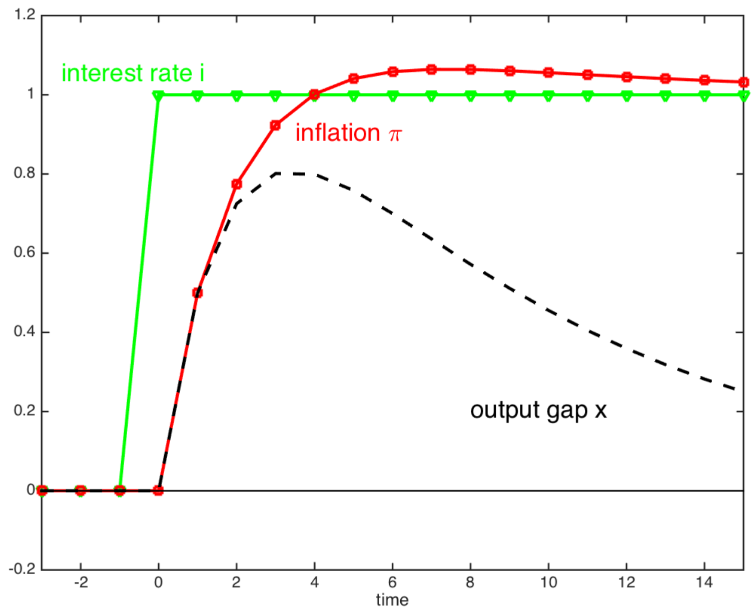


Figure 22: Impluse-response with backward-looking Phillips curve. Model $x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1})$, $\pi_t = (1-\lambda)\sum_{j=0}^{\infty}\lambda^j \pi_{t-j} + \kappa x_t$. $\sigma = \kappa = 1$, $\lambda = 0.9$.

This model also does not produce the desired temporary inflation decline.

The lesson from these exercises seems to be, that to generate a negative sign, substitution of adaptive $\pi_t^e = \pi_{t-1}$ rather than rational $\pi_t^e = E_t\pi_{t+1}$ throughout is the crucial ingredient. Neither forward-looking IS nor forward-looking Phillips curves per se are essential. However, when we do that, we return to the unstable old-Keynesian model of the first section, which does not deliver long-run stability or the long-run Fisher relationship, and is therefore inconsistent with stability at the zero bound. Nothing so far delivers a temporary negative sign and a long-run positive sign, other than the long-term-debt fiscal-theoretic model.

11 Other equilibria

There are multiple equilibria, indexed by the expectational shock $\{\delta_t\}$. As first displayed in Figure 10, one might recover a short-run negative inflation response by pairing the announcement of a rate increase with a negative multiple-equilibrium shock δ.

11.1 Multiple equilibria in the impulse-response function

Returning to the standard new-Keynesian model of equations (18)-(19) and response function displayed for $\delta_0 = 0$ in Figure 11, Figure 23 plots a range of multiple equilibrium responses to the unanticipated step function in interest rates. Each equilibrium is generated by a different choice of the expectational shock δ_0 that coincides with the monetary policy shock at date zero.

(For an impulse-response function, equilibrium choice affects only the possibility of a single shock δ_t on the date t of the announcement. An impulse-response function studies the path of $E_t\pi_{t+j},\ j = 0, 1, 2, 3...$ to an announcement made at time t. Therefore, values of $\delta_{t-j},\ j > 0$ in the solution (20) do not matter. They are the same pre- and post-announcement. Values of $\delta_{t+j},\ j > 0$ for an impulse-response function are zero after the announcement, $E_t\delta_{t+j} = 0,\ j > 0$.)

Equilibrium A has a positive additional inflation shock, $\delta_0 = 1\%$. Equilibrium B chooses δ_0 to produce 1% inflation at time 0, $\pi_0 = 1\%$. Equilibrium C chooses δ_0 to have no fiscal consequences, explained below. Between C and D lies the original fundamental equilibrium, with $\delta_0 = 0$. Equilibrium D chooses δ_0 to produce no inflation at time 0, $\pi_0 = 0$. Equilibrium E chooses $\delta_0 = -1\%$.

The figure shows graphically that the model may have too many equilibria, but all of them are stable, and all of them are Fisherian in the long run, with inflation converging to the higher nominal interest rate.

Equilibrium E verifies that the model can produce a temporary decline in inflation in response to the interest rate rise. Equilibrium E achieves that result by pairing a negative expectational or sunspot shock with the positive interest rate or expected inflation shock. We will pay a lot of attention to the plausibility of equilibrium E.

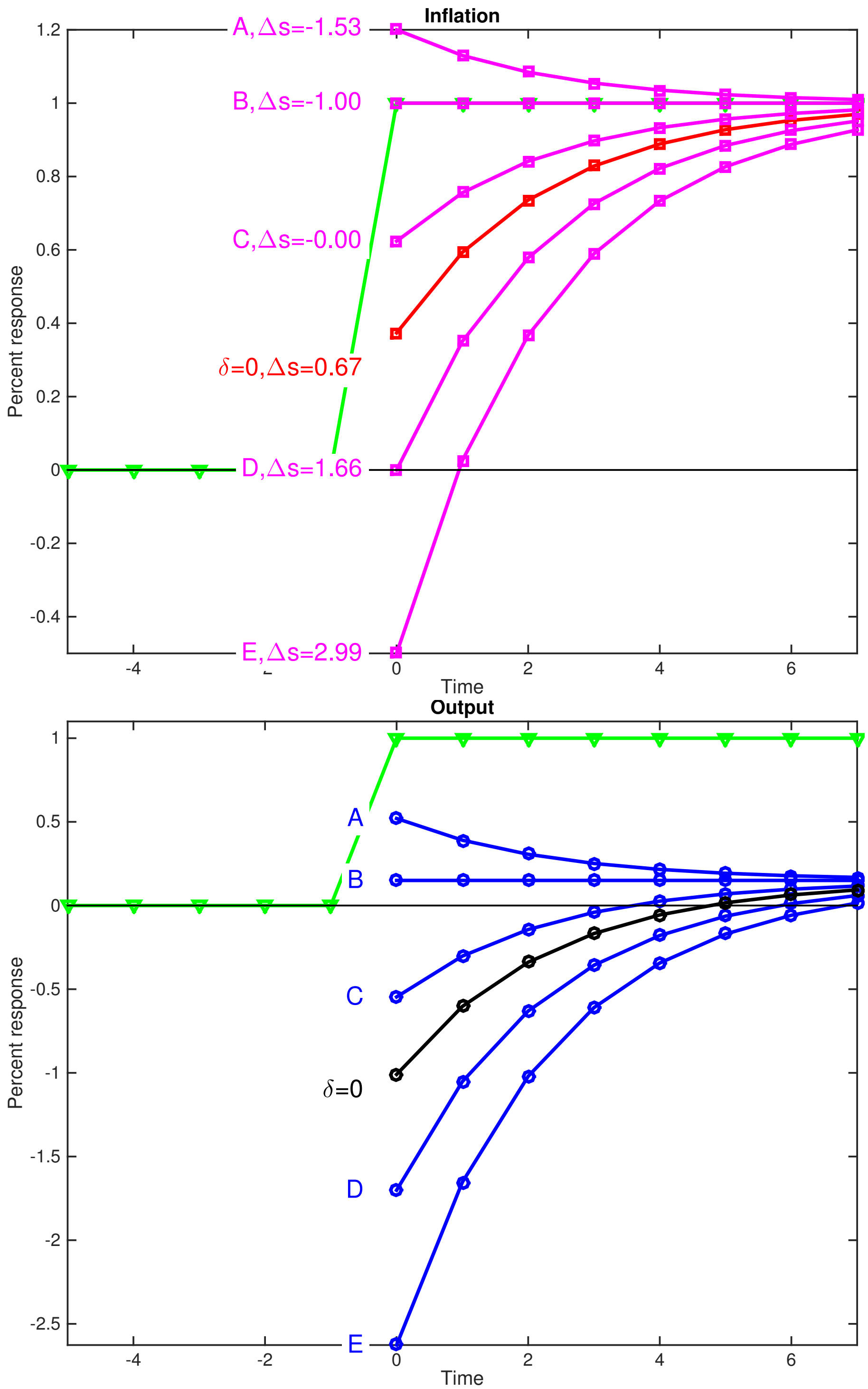


Figure 23: Multiple equilibrium responses to an unexpected interest rate rise. The solid green line gives the interest rate path. Letters identify different equilibria for discussion. The original case is $\delta_0 = 0$.

The other possibilities are informative as well. In equilibrium B inflation jumps instantly to the full increase in nominal interest rates, and stays there throughout. Output also jumps immediately to the steady-state value. Thus, despite price stickiness, the model can produce a super-neutral or super-Fisherian response, in which an interest rate rise instantly implies inflation with no output change!

Equilibrium A shows that even more inflation is possible. With a sufficiently large expectational shock, inflation can actually increase by more than the interest rate change, and then settle down, and output can increase as well.

Equilibrium D adds a small negative expectational shock δ_0, so that the initial inflation response is precisely zero. One may be troubled by inflation jumps, since inflation seems to have inertia in the data. It can be inertial in the model as well. (In continuous time, the no-jump equilibrium D is the same as the $\delta = 0$ equilibrium.)

11.2 Choosing equilibria

Is there is a convincing argument to prefer equilibria such as E, and to view this result as an embodiment of the conventional belief that raising interest rates temporarily lowers inflation?

The issue is not what shock δ_t we will see on a particular date. The question is what shock δ_t we will expect to see *on average* in response to announcements at date t of an interest rate rise.

One could make an empirical argument. But the point of this paper is to find economics for an inflation decline, not to fit the most central prediction of monetary economics through a free parameter, the correlation of expected and unexpected inflation shocks.

The rest of this section examines arguments for particular equilibrium choices.

11.3 Anticipated movements and backwards stability

The behavior of the different equilibrium choices with regard to anticipated movements is an important consideration in this choice. Most monetary policy tightenings are expected, and this kind of model describes responses to expected monetary policy changes.

Figure 10 showed in the frictionless model that mulitiple equilibria show up in the impulse-response functions of the frictionless model as a jump in inflation on the date of the announcement only. Figure 24 likewise shows multiple equilibrium responses to monetary policy change announced at $t = -3$ in the model with pricing frictions. All responses except equilibrium C are the same as in Figure 23 for $t \geq 0$. Equilibrium C is recalculated to give zero fiscal effect at time $t = -3$ rather than $t = 0$.

Figure 24 cautions us on the apparent success of equilibrium E to capture a temporary inflation decline when interest rates rise. That success relies crucially on matching the announcement of the interest rate change with the actual change. Figure 24 tells us that if a tightening is expected, as tightenings usually are, then inflation and output should both drop the most on the *announcement* of a tightening, not later when rates actually rise. (A well specified VAR should throw out such events, but analysis of policy and episodes must include them.) Classic intuition might allow small announcement effects, but the bulk of inflation and output reactions should at least coincide with if not follow actual interest rate rises.

Equilibrium D makes this announcement behavior more apparent: Inflation jumps down on the announcement but (by construction) is zero on the day of the actual rate rise, and positive thereafter.

This is an instance of a more general problematic behavior of many equilibrium choices. Most of the alternative equilibrium choices explode backwards; they imply bigger inflation shocks on the day of the announcement than at $t = 0$ when interest rates change. Equivalently, they have the property that news about events further in the future has larger effects today. The original $\delta_t = 0$ equilibrium choice has the "backwards-stable" property that it does not explode backwards, and that news about events further in the future has less and less effect today.

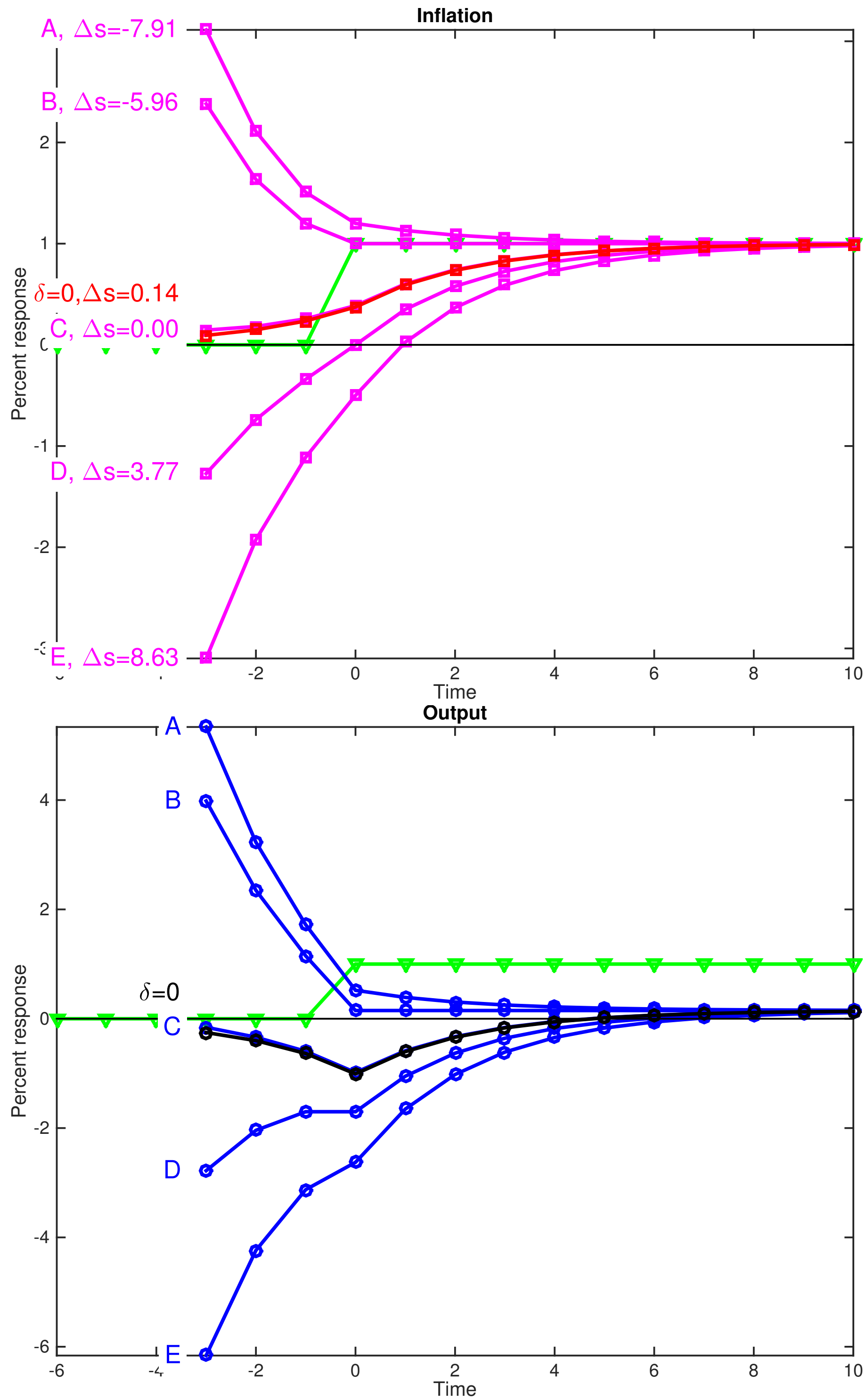


Figure 24: Multiple equilibrium responses to an anticipated interest rate change. The numbers $\Delta s =$ give the percent change in steady state surpluses required to achieve each equilibrium.

11.4 Fiscal index

Each equilibrium choice has a fiscal policy consequence. Unexpected inflation devalues outstanding nominal debt, and thus lowers the long-run financing costs of the debt. Higher real interest rates raise financing costs. For each equilibrium choice, then, I calculate the percentage amount by which long-run real primary surpluses must rise or fall for that equilibrium to emerge. That number is presented alongside the initial inflation value of each equilibrium in Figure 23 and Figure 24.

Making this calculation requires no assumption whether fiscal policy is active or passive. Even if the equilibrium choice δ is made by monetary policy or other off-equilibrium threats, and fiscal policy is completely passive, we can still calculate the surpluses that the "passive" treasury must collect. For this reason, I call the calculation a fiscal *index* – it indexes equilibria even if it does not select equilibria.

To make this calculation, I start with the valuation equation for government debt,

$$\frac{B_{t-1}}{P_t} = E_t \left[\sum_{j=0}^{\infty} \beta^j \frac{u'(C_{t+j})}{u'(C_t)} s_{t+j} \right], \tag{40}$$

where B_{t-1} denotes the face value of debt outstanding at the end of period $t-1$ and beginning of period t, P_t is the price level and s_t is the real net primary surplus.

Starting from a steady state with constant surplus s, I calculate the fractional permanent change in surplus Δs, i. e. $s_t = S^{\Delta s}$, that is required of the right hand side of expression (40) for each response function. Linearizing, I obtain in the appendix

$$\Delta s \approx -\Delta E_t (\pi_t) + \frac{1-\beta}{\sigma} \sum_{j=0}^{\infty} \beta^j \Delta E_t (x_{t+j} - x_t) \tag{41}$$

where $\Delta E_t \equiv E_t - E_{t-1}$ and t is the date of the announcement of a new policy. (The computations are nearly identical with a linearized or fully nonlinear valuation equation.)

The first term of (41) captures the fact that unexpected inflation devalues outstanding government debt. In the second term, $(x_{t+j} - x_t)/\sigma$ is the real interest rate between time t and time $t+j$. So this term captures the fact that if real rates rise, the government must pay more interest on the debt.

This calculation is simplified in many ways. I specify one-period nominal debt. Here, the objective is to focus on changes in surpluses, not the long-term debt effect studied above. A more realistic calculation adds both effects, and gives similar deviations from the responses with long-term debt plotted above. Second, in reality output changes affect primary surpluses, as taxes rise more than spending in booms and fall more than spending in recessions. Third, inflation also raises revenue due to a poorly indexed tax code. But some of these effects may represent a change in timing of surpluses – borrowing during recessions that is repaid later during booms – rather than permanent changes that affect the real value of government debt. A serious calculation of the fiscal impacts of monetary policy requires considerable detail on these lines. The point here is not quantitative realism, but to capture some of the important effects and to show how one can use fiscal considerations to evaluate different equilibrium possibilities.

The super-neutral equilibrium B in which inflation rises instantly by 1%, also marked "$\Delta s = -1.00$" in Figure 23, corresponds to a 1% decline in long-run surpluses. The 1% jump in inflation devalues outstanding nominal debt by 1%, and since output is constant after the shock there is no real interest rate change. Equilibrium A, with a larger inflation shock, corresponds to a larger than 1% decline in long-run surpluses.

Further down in Figure 23, equilibrium D has no change in inflation at time 0, and so there is no devaluation of outstanding nominal debt. However, the rise in real interest rates means that the government incurs greater financing costs. These costs require a small permanent rise in surpluses.

In between, at equilibrium C, I find the shock δ_0 that requires no change in fiscal policy at all, so $\Delta s = 0$ by construction. Here, the devaluation effect of an inflation shock just matches the higher financing costs imposed by higher real interest rates.

However, at least in this back of the envelope calibration, the difference is not large. Ignoring real interest rate effects, and discounting surpluses at a constant rate does not make a first-order difference. One does quite well grafting the simple constant-interest-rate FTPL formulas on to the new-Keynesian model.

The original equilibrium with no expectational shock, $\delta_0 = 0$, implies a small but nonzero change in surpluses, to offset the real interest rate effect.

Equilibrium E, in which inflation temporarily declines half a percentage point after the interest rate shock, requires an 1.54% rise in permanent fiscal net-of-interest surpluses. Disinflation *raises* the value of nominal debt, which must be paid.

Turning to the anticipated shocks of Figure 24, the larger inflation shocks at time $t = -3$, and the longer periods of high real interest rates, mean that the fiscal changes required to support most of the equilibria increase as we move the announcement back in time. For example, the originally super-neutral equilibrium which required a 1% decline in surpluses in Figure 23 now requires a 4.11% surplus decline, because of the larger inflation shock. And equilibrium E, selected to generate a 1% decline in inflation when interest rates rise 1%, now requires a 5.6% permanent rise in fiscal surpluses rather than 1.54%.

The exceptions to this rule are the original equilibrium choice $\delta = 0$, the equilibrium choice C or $\Delta s = 0$ with no fiscal impact, and an equilibrium (not shown) that always chooses no inflation on the announcement date. All of these equilibria have smaller fiscal impacts as interest rates are announced earlier in time, they all converge to the same point and they are all stable backward.

11.5 Using fiscal policy to choose equilibria

To produce the standard view that raising interest rates lowers inflation, we must accompany the rate rise with a fiscal tightening as in equilibrium E. Disinflation implies an unexpected present to holders of nominal government debt. Higher real interest rates also imply higher debt service payments. Fiscal authorities must be expected to raise taxes or to cut spending to make those payments.

An event such as equilibrium E is therefore a joint fiscal-monetary tightening. It provides useful guidance about what would happen if fiscal and monetary policy act together. Historic successful disinflation programs have typically combined monetary tightening with fiscal reform, which produces a rise in future (though often not current) surpluses. And unexpected monetary policy changes are often responses to unexpected economic or political conditions, which trigger fiscal policy reactions as well.

So equilibrium E of Figure 23 – or its counterpart in the announced tightening of Figure 24, where inflation drops on the mere announcement of the policy change – is compelling when one wishes to match data, understand an episode, or analyze a policy that potentially combines a monetary and a fiscal policy shock.

But our question is to evaluate the hypothetical effects of monetary policy alone. For that question, and given the possibility of any of these equilibria, it is not compelling that we should pair the monetary policy shock (rise in interest rates) with a substantial fiscal policy shock.

Monetary policy has fiscal implications, and can thereby affect inflation. Higher real rates are an inflationary force. The differences between the $\Delta s = 0$ equilibrium C, the $\delta = 0$, and in Figure 23 equilibrium D with no change in inflation capture that fact. Monetary policy raises real interest rates which impacts the budget. Equivalently, the same surpluses are discounted at a higher rate at time 0. Therefore, If fiscal surpluses s_t do not change as in C, the present value of surpluses declines, so a small inflation surprise slightly devalues outstanding debt. Equivalently, in D, for there to be no devaluation of debt, surpluses must rise a bit to pay the larger real interest costs.

This interest-expense channel is a possible fiscal-theoretic channel for the impact of monetary policy, stressed most recently by Sims (2016). However, the differences between the the $\Delta s = 0$ equilibrium C, the $\delta = 0$ equilibrium, and the no-inflation-jump equilibrium D are small here, and get smaller for anticipated policy. In addition, the interest-expense channel C raises inflation, so does not even directionally help in the quest for a temporary negative response.

The fiscal calculation can serve as an equilibrium-selection mechanism. The fiscal theory of the price level and the definition that a "monetary policy shock" means one with no fiscal policy response $\Delta s = 0$, selects equilibrium C.

However, one can also keep a passive-fiscal view of these calculations. In that case, the fiscal index merely reveals how much fiscal policy must "passively" adjust to whichever equilibrium is selected by some other means. One can then decide if the required fiscal adjustment is reasonable or not. If criterion x selects a path that requires the "passive" fiscal authority to raise 200% of GDP in taxes, it's not going to happen.

In this vein, one might argue for equilibria that have limited or small fiscal requirements, rather than equilibria which require large changes in surpluses to be generated by the "passive" fiscal authorities, or insist on equilibria with exactly zero fiscal implications. That argument, which we might call fiscal theory lite, puts us in a range around the $\Delta s = 0$ equilibrium, and still limits our ability to produce disinflation.

Pushing the announcement date back as in Figure 24 enlarges these fiscal considerations. The equilibria that are not backward-stable all have larger and larger fiscal policy consequences as the announcement is pushed back. Conversely, $\delta = 0$ equilibrium, choice, the no fiscal impact $\Delta s = 0$ choice and the no inflation jump choice converge as the announcement is pushed back. By an announcement $t = -3$ shown in Figure 24, the $\delta = 0$ equilibrium and the $\Delta s = 0$ equilibrium are already visually indistinguishable.

Equilibria with no jump in inflation are also attractive. Equilibrium D in Figure 23 has this property, and one can construct an equilibrium with no change in inflation upon announcement for the $t = -3$ shock of Figure 24. We do not see inflation jumps in the data, and new-Keynesian models are often specified so that inflation must be set one or more periods in advance. This choice also is stable as the announcement horizon moves backward.

In sum, the principles of small fiscal requirements, sensible behavior as announcements come earlier than actual rate changes, or limited jumps in inflation all push one to the view that equilibria near the original $\delta = 0$ equilibrium are sensible, and the others less so. One does not need to be a fiscal theory of the price level zealot for this choice.

The absence of the affirmative is more important here than the negative. I have not found a strong economic reason that we *should* pair large negative expectational, equilibrium-changing, or fiscal-policy-induced shocks δ_0 with announcements of interest rate rises. I find no economic mechanism for producing a large unexpected inflation shock, except fiscal policy, which suggests no such shock when thinking about monetary policy interventions. The essential definition of a central bank is that it can rearrange government debt but cannot take fiscal action such as a helicopter drop. So, this discussion leads me to look away from paths such as equilibrium E as the device to generate a temporary decline in inflation when interest rates rise, and to look elsewhere.

12 Taylor rules

Taylor rules with active responses to inflation are usually invoked to prune equilibria, and to deliver a short-run negative inflation response. Can writing policy in terms of a Taylor rule help us to choose among the equilibria displayed in Figures 23 and 24?

12.1 Constructing Taylor rules

As previewed above, the solution method using equations (20)-(21) does not assume a peg. We can *construct* a Taylor rule that supports any of the equilibria displayed in Figures 23 and 24, as follows. Assume interest rate policy is

$$i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) \tag{42}$$

where i_t^* is the step-function or other desired equilibrium interest rate path, π_t^* is the equilibrium path of inflation, i.e. one of the choices displayed in Figure 23 or Figure 24, and ϕ_π is arbitrary. If $\phi_\pi > 1$, then the desired path $\{i_t^*,\ \pi_t^*,\ x_t^*\}$ is the unique locally-bounded (nonexplosive) equilibrium. (Here, of course, I consider the case away from the zero bound. At the zero bound, one must construct more complex promises of future Taylor rules as discussed briefly in section 4.3.)

Traditionally, one solves this kind of model by adding to the IS and Phillips curves (18)-(19) a monetary policy rule, say

$$i_t = \hat{i}_t + \phi_\pi \pi_t \tag{43}$$

and then solving for equilibrium $\{i_t,\ \pi_t,\ x_t\}$ given shocks including $\hat{i}_t$. To produce an impulse-response function, as I have, one must find a monetary policy disturbance sequence $\{\hat{i}_t\}$ that produces the desired response of equilibrium interest rates $\{i_t\}$. In general, since $\phi > 0$, the disturbance sequence $\{\hat{i}_t\}$ is different from the interest-rate response.

However, equations (42) and (43) are the same for

$$\hat{i}_t = i^*_t - \phi_\pi \pi^*. \tag{44}$$

In this context, then, my procedure – solving for output and inflation given the desired equilibrium interest rate path, and then constructing monetary policy that supports the desired equilibrium by (43), or by (44) – amounts simply to a way to avoid the unpleasant search for the monetary policy shock disturbance $\{\hat{i}_t\}$ that produces the desired equilibrium interest rate path. This clever approach and interpretation is due to Werning (2012).

Expressing the Taylor rule as in (42) emphasizes that the active Taylor rule includes two policy settings. The rule consists of an interest rate target, $\{i^*_t\}$, and an equilibrium-selection rule, the choice of ϕ_π and $\{\pi^*_t\}$ from the set of equilibrium $\{\pi_t\}$ consistent with the interest rate target. The interest rate target determines the path of equilibrium interest rates. The selection rule specifies a set of off-equilibrium threats or beliefs, that rules out all but the desired equilibrium path of inflation. (Many other equilibrium selection schemes achieve the same purpose, for example see Atkeson, Chari, and Kehoe (2010) and the discussion in the online appendix to Cochrane (2011).)

This construction (42) and its equivalence with (43) addresses the first question: Does the assumption of a Taylor rule solve the equilibrium selection problem? No. Via (42), all of the equilibria, any choice of δ_0 such as graphed in Figure 23 and Figure 24, are consistent with an active Taylor rule, and equation (42) shows how to construct the Taylor rule assumption, down to the specific shock sequence $\hat{i}_t$ – that generates any desired equilibrium. The fact of adding a Taylor rule, by itself, doesn't help us at all to choose among equilibria.

12.2 Open-mouth policy

Furthermore, if the Fed can induce jumps δ_t by equilibrium-selection policy implemented by an active Taylor rule $\phi_\pi > 1$, there is no reason for the Fed to bother with interest-rate policy. Why pair the equilibrium-selection policy that shifts unexpected inflation downwards with a rise in interest rates that shifts expected inflation upwards? If the Fed wants to induce temporarily lower inflation, all it need to is to announce a new inflation target.

To be specific, suppose the Fed follows a Taylor rule

$$i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*). \tag{45}$$

Suppose the Fed, starting at $i_t^* = 0,\ \pi_t^* = 0$ for $t < 0$, leaves i_t^* alone, but shocks monetary policy for $t \geq 0$ to

$$\pi_t^* = \delta_0 \lambda_1^{-t}. \tag{46}$$

Here, δ_0 is a constant indexing how large the monetary policy shock will be. This is a pure, temporary, change in the Fed's inflation target.

Equivalently, suppose the Fed follows a Taylor rule

$$i_t = \hat{i}_t + \phi_\pi \pi_t. \tag{47}$$

Suppose that the Fed, starting at $\hat{i}_t = 0$ for $t < 0$, shocks monetary policy for $t \geq 0$ to

$$\hat{i}_t = -\delta_0 \phi_\pi \lambda_1^{-t}. \tag{48}$$

This is a pure, temporary, monetary policy disturbance.

Figure 25 plots the responses of inflation and output to these monetary policy disturbances. Inflation and output move, but interest rates are constant throughout the episode.

Intuitively, in response to a shock $\hat{i}_0$, and its expected subsequent values $\{\hat{i}_t\}$, inflation jumps down just enough so that the systematic component of policy in (47) exactly offsets the shock, and the actual interest rate i_t does not change at all. In response to the shock π_0^*, and to the expected subsequent values $\{\pi_t^*\}$, inflation jumps to $\pi_0 = \pi_0^*$. Via the Taylor rule (45), this change in inflation is is just enough so that actual interest rates do not change.

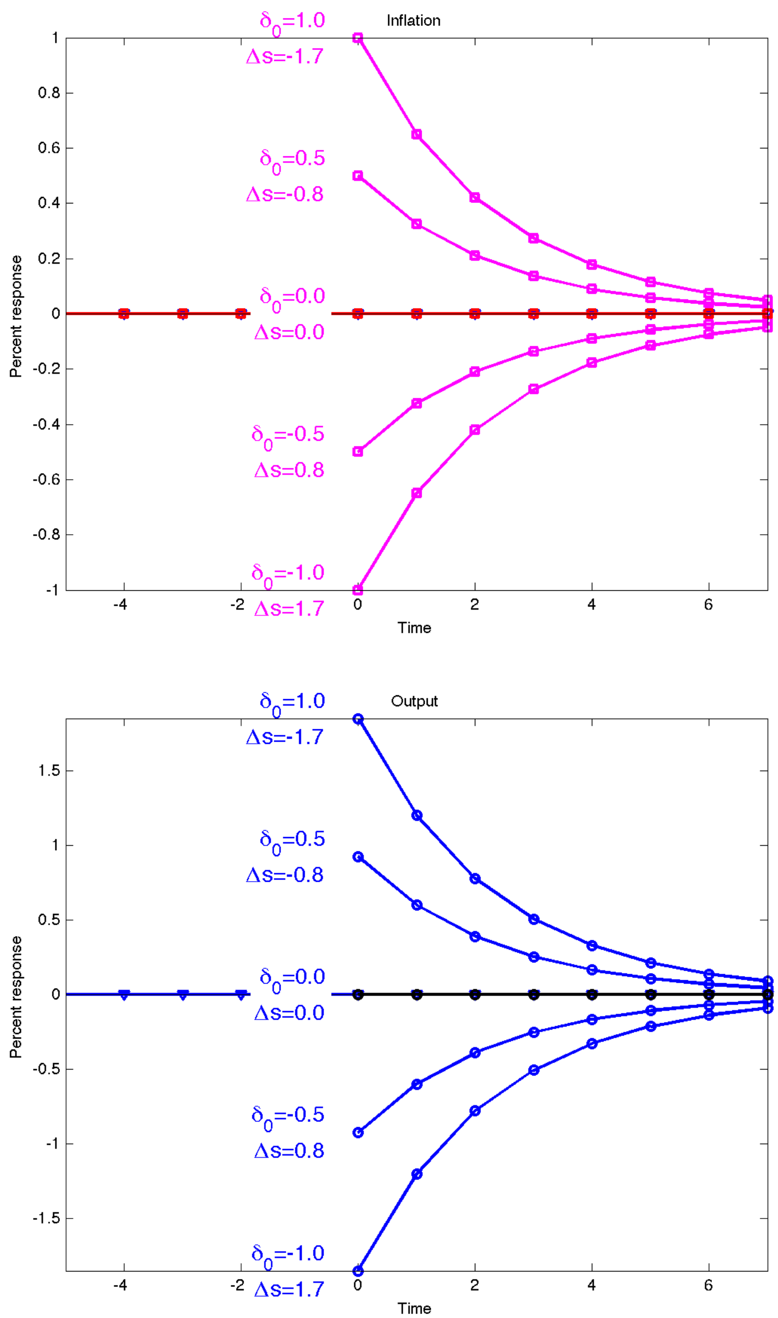


Figure 25: Response of inflation and output to a shift in inflation target with no shift in interest rate target.

Once we see how pure open-mouth / equilibrium-selection operations can induce the shocks δ_0, pairing shocks to inflation as in Figure 25 with shocks to interest rates is arbitrary.

Now, perhaps this is our world. Monetary policy at the zero bound has seemed to evolve into central banker statements accompanied by no actual changes in interest rates or asset purchases. Reserve Bank of New Zealand Governor Donald Brash (Brash (2002)) coined the term "open-mouth operations" observing that he seemed to be able to move interest rates by simply talking, without conducting open market operations. The open mouth operation described by Figure 25 is doubly removed from action, since the central bank can apparently move inflation without even moving interest rates.

Perhaps inflation really has little to do with economics; supply and demand, intertemporal substitution, money, and so forth. Perhaps inflation really is predominantly a multiple-equilibrium question. Perhaps "monetary" policy affects inflation entirely by government officials making statements, with implicit never-observed off-equilibrium threats, that cause jumps from one equilibrium to another, validated by passive fiscal policy. Perhaps the analysis of monetary policy should go back where it left off in the 1950s and 1960s, in which inflation was largely thought to comprise "wage-price spirals," and inflation policy centered on talk not action. Perhaps changes to actual interest rates, though economically irrelevant and even counterproductive in the long run, evolved as some sort of communication and signaling equilibrium to indicate a policy shock.

If so, again, sufficient becomes necessary. The quest of this paper – a simple, transparent, baseline economic model of the effect of interest rates on inflation – is over, with a negative result and a disquieting implication for the status of monetary policy in the arsenal of robust and well-understood phenomena.

Figure 25 includes the change in long-run surpluses needed to validate each equilibrium. I include this number as a reminder that it is there. If one takes the fiscal-passive view, these are the resources that the "passive" Treasury will need to come up with to validate the Fed's "active" equilibrium-selection policy.

If one takes the fiscal theory of the price level view, these calculations have a much simpler interpretation. In this case, the indicated change in expected future surpluses results in one of these equilibria as the unique equilibrium. These are movements in inflation achievable by a pure change in *fiscal* policy, when monetary policy leaves nominal interest rates unchanged. In the fiscal theory of the price level, the Fed still sets *expected* inflation freely by setting nominal interest rates, while fiscal policy uniquely chooses *unexpected* inflation, and hence δ_0.

If one takes a passive-fiscal view, nonetheless the fiscal authority must passively come up with these surpluses to validate monetary policy. The Fed announces a policy shock. Actual interest rates don't move. Inflation jumps. People expect the fiscal authority to loosen in response to the announced monetary policy shock. To an economist who cannot see the shock announcement, the whole affair looks exactly like a sunspot. If this story challenges believability, that is the point.

12.3 Reasonable disturbances

Perhaps, however, expressing policy in terms of a traditional single shock (43) $i_t = \hat{i}_t + \phi_\pi \pi_t$, rather than in terms of a separate interest rate target i_t^*, inflation target π_t^* and an equilibrium selection policy in (42) $i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*)$, via (44) $\hat{i}_t = i_t^* - \phi_\pi \pi^*$, will indicate that one or another equilibrium results from a more sensible monetary policy disturbance.

Figure 26 gives the monetary policy disturbance $\hat{i}_t$ in a Taylor rule $i_t = \hat{i}_t + 1.5\pi_t$ needed to produce the step function rise in equilibrium interest rates and each of the possible inflation outcomes (equivalently, equilibrium choices δ) from Figures 23 (an unexpected interest rate rise, solid) and 24 (an expected interest rate rise, dashed).

The equilibria with large positive inflation shocks such as A result from negative monetary policy disturbances, and vice versa. When i and π move together, $\phi > 1$ in $i_t = \hat{i}_t + \phi_\pi \pi_t$, means $\hat{i}_t$ must go the opposite way. Again we see the interesting property of the standard explosive Taylor rule that interest rate rises correspond to negative policy shocks.

All the disturbances end up at $\hat{i}_t = -0.5$, since they all end up with $i_t = 1$ and $\pi_t = 1$, and $1.0 = -0.5 + 1.5 \times 1.0$.

To produce the baseline $\delta_0 = 0$ inflation pattern in the unanticipated (solid) case,

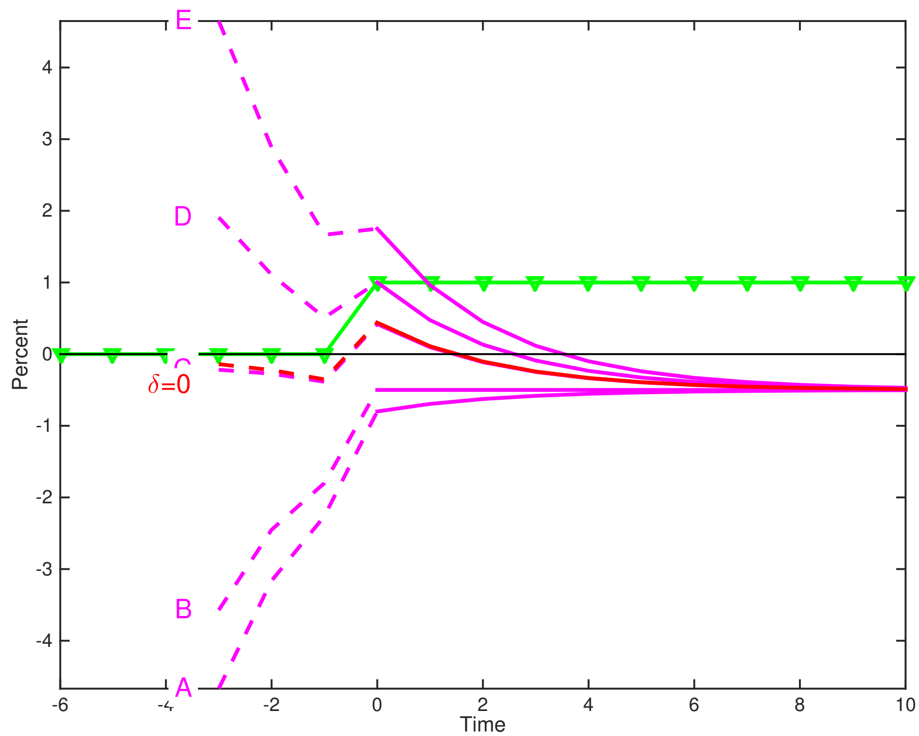


Figure 26: Monetary policy disturbance $\hat{i}_t = i^*_t - \phi_\pi \pi^*_t$ that produces each equilibrium with Taylor parameter $\phi_\pi = 1.5$. Dashed lines give values for the announcement at time $t = -3$. Solid lines give values for the announcement at the same time as the rate rise at time $t = 0$. Letters and $\delta = 0$ correspond to the equilibria shown in previous figures.

the disturbance $\{\hat{i}_t\}$ follows a pattern with geometric decay. This pattern mirrors the geometric rise of inflation, relative to the step function rise in observed interest rates.

To produce a transitory decline in inflation, one needs the larger disturbance of equilibrium E. E is a strong positive disturbance that slowly melts away to negative in a geometric pattern. Comparing the two $\{\hat{i}_t\}$ it's hard to say one is a lot more reasonable than the other. Both disturbances are first positive and then negative.

The dashed lines, showing the monetary policy disturbances necessary to produce the responses to an *anticipated* rise in equilibrium interest rates are wilder. Viewed through the lens of a Taylor rule, the Fed does not simply announce that rates will rise in the future. All of these equilibria feature inflation that moves between the announcement and the actual interest rate rise. Therefore, with $i_t = \hat{i}_t + \phi \pi_t$, if π_t moves and i_t

does not move, the Fed must have a disturbance $\hat{i}_t$ that offsets inflation $\phi\pi_t$. As in the open-mouth operations, the Fed announces a monetary policy shock $\{\hat{i}_t\}$. Inflation moves so much, however, that the systematic component of monetary policy $\phi_\pi\pi_t$ exactly offsets the monetary policy shock $\{\hat{i}_t\}$, producing a change in inflation with no change at all in the actual interest rate.

For the anticipated rate rise (dash), there is no equilibrium in which $\hat{i}_t$ does not move ahead of the actual interest rate rise, as there is no equilibrium in which inflation does not move ahead of an anticipated interest rate rise. But the baseline equilibrium $\delta = 0$ and the equilibrium C with no fiscal consequence $\Delta s = 0$ at least have disturbances $\hat{i}_t$ that are small and that decline as the policy announcement moves back in time. By contrast, the disturbance E is large and grows as the announcement time moves back.

In sum, if one pursues "reasonable" specifications for the monetary policy disturbance, in the context of a Taylor rule of the form $i_t = \hat{i}_t + \phi_\pi\pi_t$, as an equilibrium selection device, that path does not strongly suggest equilibria such as D and E in which inflation declines temporarily. In fact, the view that the Fed makes big monetary policy shocks that induce big changes in inflation, which through the systematic component of policy $\phi_\pi\pi_t$ then just offset the monetary policy shock and produce no change in interest rate, may seem the more far-fetched assumption.

12.4 The standard three-equation model

The open-mouth calculation of the last section is a special case of the standard analysis of a monetary policy shock in the three-equation new-Keynesian model. Didn't that model produce the desired sign, one might ask?

I repeat here the standard equations,

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1}) \tag{49}$$

$$\pi_t = \beta E_t\pi_{t+1} + \kappa x_t \tag{50}$$

$$i_t = \phi\pi_t + v_t^i \tag{51}$$

$$v_{t+1}^i = \rho v_t^i + \epsilon_{t+1} \tag{52}$$

Figure 27 plots the response of inflation and interest rates to an unexpected monetary policy shock v_t^i for this model.

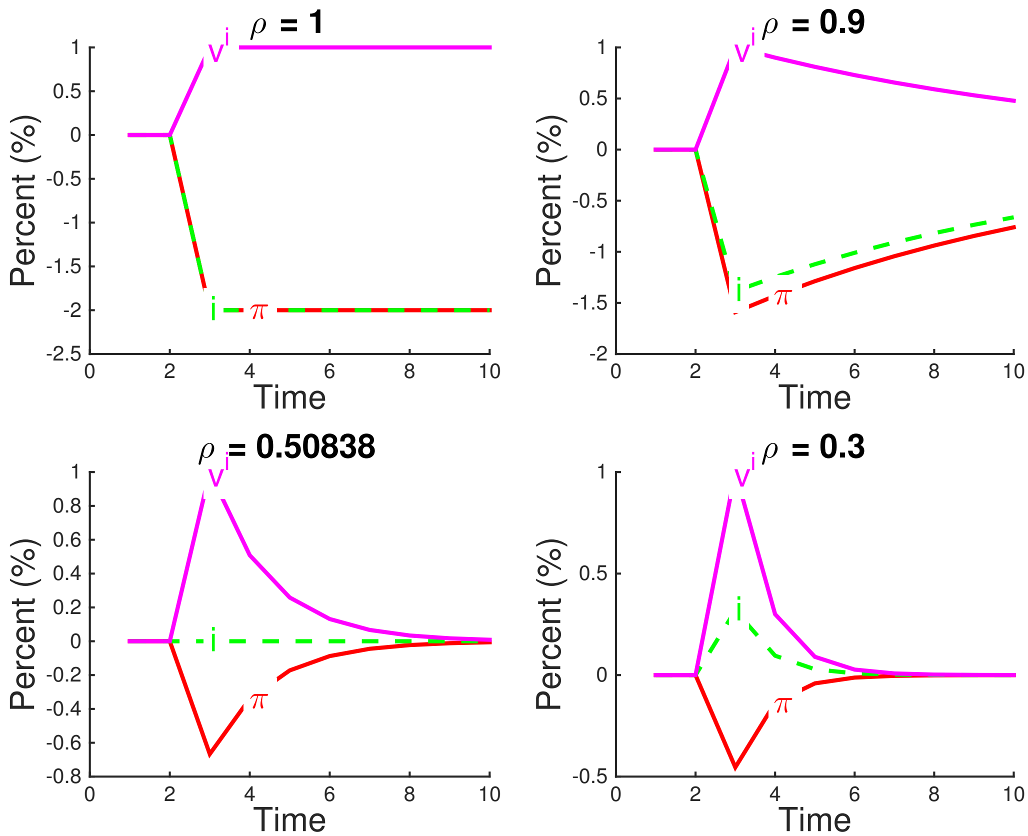


Figure 27: Response of inflation and interest rates to an AR(1) monetary policy shock v^i with persistence ρ in the standard three-equation new-Keynesian model. $\beta = 0.95$, $\kappa = 1/2$, $\sigma = 1$, $\phi = 1.5$.

The top left panel plots the response to a permanent shock, as in the other figures. The standard three equation model is perfectly Fisherian in this case! The positive monetary policy *shock* v^i produces a negative response of actual interest rates and inflation i and π. The rule $i_t = \phi\pi_t + v_t^i$ becomes $-2 = 1.5(-2) + 1$. This example emphasizes the important difference between monetary policy *shocks* and equilibrium interest rates. The shock is positive, but interest rates fall. One might opine that it is a strange "tightening" by which actual interest rates fall one for one with inflation.

The top right panel plots the response to a very persistent $\rho = 0.9$ shock. Here the inflation and interest rate responses are easier to distinguish. Still, interest rates fall, and one sees a largely Fisherian result. The endogenous negative $\phi\pi$ part of interest rates still overwhelms the positive monetary policy shock.

The bottom left panel plots the response in the knife-edge case $\rho = 1/\lambda_1$ that the monetary policy shock becomes an "open-mouth" operation. Here the endogenous effect $\phi\pi_t$ just offsets the shock v_t^i so inflation moves with no change at all in interest rates.

Finally, the bottom right panel shows that for a sufficiently short-lived shock, $\rho = 0.3 < 1/\lambda_1$, the shock v^i exceeds the endogenous response $\phi\pi$, so a positive monetary policy shock increases interest rates i and decreases inflation π.

Is this, at last, the answer we are looking for? It is not a fundamentally different mechanism than the cases analyzed so far. It combines a swiftly mean-reverting process for the interest rate, as graphed in Figure 12, with a strong contemporaneous fiscal contraction like case E of Figure 23. Repeating for convenience the formula (20) for the general inflation solution,

$$\pi_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left[i_t + \sum_{j=1}^{\infty}\lambda_1^{-j} i_{t-j} + \sum_{j=1}^{\infty}\lambda_2^{j} E_{t+1} i_{t+j}\right] + \sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j} \tag{53}$$

we can see the case. If interest rates i mean-revert quickly enough, the central terms will be small. Then, if we add a large enough δ shock at time zero, we produce a negative inflation response.

So the standard three equation model, with sufficiently transitory monetary policy shock to produce the "right" sign, is really just a case of a large multiple-equilibrium δ shock, with a strong "passive" fiscal tightening.

13 Other modeling directions; necessary vs. sufficient assumptions

We have searched for a simple modern, rational economic model, consistent with the observed stability and quiet behavior of inflation at the zero bound, that restores the traditional view in which a rise in interest rates produces at least a temporary decline in inflation. The result is, so far, largely negative. Price stickiness, money, backward looking Phillips curves, alternative equilibrium choices and active Taylor rules do not provide a convincing basis to overturn the short-run Fisherian predictions of the frictionless model. They do not begin to overturn its long-run Fisherian prediction.

The one model that works is the fiscal theory with long-term debt, in which an interest rate rise amounts to a swap of current for future inflation. However, the temporary decline in inflation in this model is a deeply fiscal-theoretic phenomenon, is a sensitive result of particular expected fiscal policies, only applies to unexpected rate rises, and does not rescue traditional policy advice, either on how to engineer a disinflation or that the Fed can and should systematically raise interest rates in response to inflation.

The next directions one might go to reestablish the conventional view involve abandoning one of the qualifiers simple, modern, or economic.

In order to produce the standard signs, one might add ingredients to micro-found the ad-hoc old-Keyensian model's basic structure, to try to restore the "modern" and "economic" adjectives, while also somehow repairing the model's prediction that the zero bound is unstable. One might add extensive borrowing or collateral constraints, hand-to-mouth consumers, irrational expectations or other irrational behavior, a lending channel, or other frictions, continuing the 60-year old quest to undermine the permanent income hypothesis.

Gabaix (2016) is a concrete behavioral example: By severely downweighting $E_t x_{t+1}$ in the IS curve, (18), Gabaix produces traditional explosive dynamics and therefore a negative sign. His model still needs patching to match the long run and the zero bound episode – the key observation that rules out explosive dynamics – but it represents a state-of-the-art example of the possibility. (See Cochrane (2016a).)

The models in this paper are also quite simple by the standards of calibrated or estimated new-Keynesian models, such as Smets and Wouters (2003), Christiano, Eichenbaum, and Evans (2005) and their descendants. García-Schmidt and Woodford (2015) and Angeletos and Lian (2016) (see also the extensive literature review in the latter) fundamentally change the nature of equilibrium and expectations. Perhaps adding the range of complexity in such models – habits, labor/leisure, production, capital, variable capital utilization, adjustment costs, alternative models of price stickiness (despite my unsuccessful foray into ad-hoc Phillips curve lags) or informational, market, payments, monetary, and other frictions, or fundamentally different views of expectations and equilibrium – will perturb dynamics in the right way.

But following these paths abandons the qualifiers "simple" or "economic." They mean that more complex or non-economic ingredients are *necessary* as well as *sufficient* to deliver the central result. Doing so admits that there is no simple, rational economic model one can put on a blackboard, teach to undergraduates, summarize in a few paragraphs, or refer to in policy discussions to explain at least the signs and rough outlines of the operation of monetary policy–nothing like, say, the stirring and simple description of monetary policy in Friedman (1968).

Such an intellectual outcome would be unusual in macroeconomics. The standard new Keynesian approach views the complex models or even behavioral modfications as refinements, building on (18)-(19). The refinements help to match the details of model dynamics with those observed in the data, but the simple model captures capturing the basic message, signs, stability and intuition. The standard real business cycle approach views complex models as refinements, building on the stochastic growth model, but that simple model can still capture the basic story. The large multi-equation Keynesian models developed in the 1970s built on simple ISLM models to better match details of the data, but modelers felt that the simple ISLM model captured the basic signs and mechanisms.

This was all healthy. Economic models are quantitative parables, and one rightly distrusts predictions that crucially rely on the specific form of frictions, especially frictions that have little microeconomic validation.

So, if we go down the route, that much greater complexity or abandonment of simple economics, rational behavior and expectations, are *necessary* as well as sufficient to generate the basic sign and stability properties of monetary policy, we have already admitted an important defeat. The scientific basis on which we analyze policy, and its advise its use to public officials and public at large, is clearly much more tenuous.

14 VARs

If theory and experience point to a positive reaction of inflation to interest rates, perhaps we should revisit the empirical evidence behind the standard view to the contrary.

The main formal evidence we have for the effects of monetary policy comes from vector autoregressions (VARs). There are several problems with this evidence.

First, the VAR literature almost always pairs the announcement of a new policy with the change in the policy instrument, i.e. an unexpected shock to interest rates. That habit makes most sense in the context of models following Lucas (1972) in which only unanticipated monetary policy has real effects, and in the context of regressions of output on money, rather than interest rates, in which VARs developed (Sims (1980)).

But in the models presented here, anticipated monetary policy has strong effects. In the world, most monetary policy changes are anticipated. VARs may still want to find rare unexpected rate movements, as part of an identification strategy to find changes in policy that are not driven by changes of the Fed's expectations of future output and inflation. But there is no reason, either theoretical, empirical, historical, or for policy-relevant analysis, to focus all model analysis on a surprise exogenous rate change. It's like evaluating a Ferrari by driving it around the parking lot. OK, it should drive correctly around the parking lot, but it should do a lot else as well. That mismatch drives the less formal style of analysis in this paper – matching model predictions with broad facts about the data.

Furthermore, the models with money presented here, as in Figure 17, only had a chance of delivering the standard inflation decline if the interest rate rise was anticipated. An empirical technique that isolates unexpected interest rate rises cannot find or verify that theoretical prediction.

Second, the analysis of multiple equilibria in Figure 23 and Figure 24 found that inflation declines occur when an interest rate rise is paired with a fiscal policy tightening. Now, all interest rate changes are responses to *something*. So it is plausible that whatever motivates the Fed to raise or lower interest rates also motivates fiscal authorities to change course, even if those events are truly uncorrelated with expected output or inflation and thus candidate VAR shocks. It is plausible that rate shocks in our data set are like equilibrium E of Figure 23. VARs have to date made no attempt to orthogonalize monetary policy shocks with respect to fiscal policy, especially expected future fiscal policy which is what matters here. But such a VAR estimate does not measure the effects of a pure monetary policy shock, which is our question.

Third, VARs typically find that the interest rate responses to an interest rate shock are transitory, as are those of Figure 12. As a result, they provide no evidence on the long-run response of inflation to permanent interest rate increases.

Fourth, and most of all, the evidence for a negative sign is not strong, and one can read much of the evidence as supporting a positive sign. From the beginning, VARs have produced *increases* in inflation following increases in interest rates, a phenomenon dubbed the "price puzzle" by Eichenbaum (1992). A great deal of effort has been devoted to modifying the specification of VARs so that they can produce the desired result, that a rise in interest rates lowers inflation.

Sims (1992), studying VARs in five countries notes that "the responses of prices to interest rate shocks show some consistency - they are all initially positive." He also speculates that the central banks may have information about future inflation, so the response represents in fact reverse causality. Christiano, Eichenbaum, and Evans (1999) took that suggestion. They put shocks in the order output (Y), GDP deflator (P), commodity prices (PCOM), federal funds rate (FF), total reserves (TR), nonborrowed reserves (NBR), and M1 (M) (p. 83). Their idea is that commodity prices capture information about future inflation that the Fed may be reacting to, so including commodity prices first isolates policy shocks that are not reactions to expected inflation. This ordering also assumes that policy cannot affect output, inflation, or commodity prices for a quarter. With this specification (their Figure 2, top left), positive interest rate shocks reduce output. But even with the carefully chosen ordering, interest rate increases have no effect on inflation for a year and a half. The price level then gently declines, but remains within the confidence interval of zero throughout. Their Figure 5, p.100, shows nicely how sensitive even this much evidence is to the shock identification assumptions. If the monetary policy shock is ordered first, prices go up uniformly. The inflation response in Christiano, Eichenbaum, and Evans (2005) also displays a short run price puzzle, and is never more than two standard errors from zero.

Even this much success remains controversial. Hanson (2004) points out that commodity prices which solve the price puzzle don't forecast inflation and vice versa. He also finds that the ability of commodity prices to solve the price puzzle does not work after 1979. Sims (1992) was already troubled that commodities are usually globally traded, so while interest rate increases seem to lower commodity prices, it's hard to see how that could be the effect of monetary policy.

Ramey (2015) surveys and reproduces much of the exhaustive modern literature. She finds that "The pesky price puzzle keeps popping up." Of nine different identification methods, only two present a statistically significant decline in inflation, and those only after four or more years of no effect have passed. Four methods have essentially no effect on inflation at all, and two show strong, statistically significant positive effects, which start without delay. Strong or reliable empirical evidence for a short-term (within 4 years) negative inflation effect is absent in her survey.

The Christiano, Eichenbaum, and Evans (1999) procedure may seem fishy already, in that so much of the identification choice was clearly made in order to produce the desired answer, that higher interest rates lead to lower inflation. Nobody spent the same amount of effort seeing if the output decline represented Fed reaction to news, because the output decline fit priors so well. As Uhlig (2006) points out, however, that procedure is defensible. *If* one has strong theoretical priors that positive interest rate shocks cause inflation to decline, then it makes sense to impose that view as part of shock identification, in order to better measure that and other responses. (Uhlig's eloquent introduction is worth reading and contains an extensive literature review.)

But imposing the sign only makes sense when one has that strong theoretical prior; when, as when these papers were written, existing theory uniformly specified a negative inflation response and nobody was even considering the opposite. In the context of this paper, when theory specifies a positive response, when only quite novel theories produce the negative sign, and we are looking for empirical evidence on the sign, following identification procedures that implicitly or explicitly throw out positive signs does not make sense. And even imposing the sign prior, Uhlig like many others finds that "The GDP price deflator falls only slowly following a contractionary monetary policy shock."

With less strong priors, positive signs are starting to show up. Belaygorod and Dueker (2009) and Castelnuovo and Surico (2010) find that VAR estimates produce positive inflation responses in the periods of estimated indeterminacy. Belaygorod and Dueker (2009) connect estimates to the robust facts one can see in simple plots: Through the 1970s and early 1980s, federal funds rates clearly lead inflation movements. Dueker (2006) summarizes.

And of course all of this evidence comes from the period in which the Fed kept a very small amount of non-interest-bearing reserves, the money multiplier plausibly bound, and the Fed implemented interest-rate changes by changing the small quantity of reserves or promising to do so. To state that interest rates in this regime have the same effect on inflation as they do in the current regime, in which the Fed will change interest rates by changing the interest it pays on super-abundant reserves, with no open market operations at all, requires theory.

15 Fiscal policy anchoring and warning

In several places I have advocated fiscal theory of the price level arguments. I collect here quick answers to some of the most common objections to these uses of fiscal arguments. Cochrane (2005) contains many more.

15.1 Exogenous surpluses?

The fiscal theory of the price level does not require that surpluses are "exogenous," or set independently of monetary policy or economic conditions. The government debt valuation equation

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} s_{t+j} \tag{54}$$

is an equilibrium condition; it works in exactly the same way as the standard asset valuation formula in which the price per share is the present value of dividends d,

$$p_t = \frac{\text{Value}_t}{\text{Shares}_t} = \sum_{j=0}^{\infty} \beta^j \frac{u'(c_{t+j})}{u'(c_t)} d_{t+j},$$

or the permanent income formula in which consumption is the present value of future income

$$c_t = rk_t + r\beta E_t \sum_{j=0}^{\infty} \beta^j y_{t+j}$$

In all cases, though it is often useful to think through the logic of models with a story that price level, stock price, or consumption, are determined "given" the quantities on the right-hand side, in fact all quantities are economically endogenous and co-determined. Furthermore, this analogy should but to rest any concern that the first equation is a "budget constraint" or that it involves scary off-equilibrium threats, any more than do the latter two.

The government debt valuation equation holds in equilibrium in all regimes – "active" or "passive." Therefore active and passive regimes are observationally equivalent. There is no test for Granger causality, no regression of surpluses on debt or interest rates on inflation, no relationship among time series drawn from the equilibrium of an economy, that can distinguish fiscally active and passive, or monetary active and passive (using the terminology in Leeper (1991)) regimes. That fact drives the style of analysis in this paper – only by looking across regimes, or from experiments such as the zero bound episode, and asking for Occam's razor simplicity can we tell theories apart.

15.2 On and off equilibria

If a government pays back its debts, then the regime must be passive and the fiscal theory cannot operate, no? No.

The key to a passive regime is that the government raises surpluses to pay back debts arising from *any* cause. For example, if a bubble or sunspot causes the price level to decline 50%, the government will double taxes to repay that windfall to bondholders, just as it would to repay debt incurred to finance a war.

Furthermore, the distinction between on equilibrium and off equilibrium responses common in the new-Keynesian Taylor-principle view applies to the fiscal theory as well. The government can commit to repaying any "on-equilibrium" debt, but defaulting or inflating away "off-equilibrium" debt. Following King (2000), I wrote in (42) the Taylor rule in a new-Keynesian model as

$$i_t = i_t^* + \phi_\pi(\pi_t - \pi_t^*) \tag{55}$$

This observationally equivalent form allows us to separate how a central bank's observed interest rate i_t^* responds to observed equilibrium inflation π_t^* from how the the central bank threatens to modify interest rates off-equilibrium $(i_t - i_t^*)$ from off-equilibrium inflation $(\pi_t - \pi_t^*)$. This writing of the Taylor rule makes it clear that no data from an equilibrium can enlighten us about the crucial off-equilibrium threat, how the central bank would respond to a sunspot inflation to rule it out.

Precisely the same argument applies to the fiscal theory. If surpluses respond to debt levels,

$$s_t = \hat{s}_t + \alpha(B_{t-1}/P_t) \tag{56}$$

then, yes, fiscal policy is "passive," surpluses adjust on the right hand side of (54) for any price level, and the equation drops out of equilibrium determination. But if we write the fiscal rule in form analogous to (55)

$$s_t = \hat{s}_t + \alpha(B_{t-1}/P_t^*) \tag{57}$$

where $*$ denotes the government's desired equilibrium, then we see exactly the same response in equilibrium – the government pays off its debts – but no response to off-equilibrium price level bubbles or sunspots. As Taylor-rule off-equilibrium interest rate rises select a unique equilibrium, so fiscal-rule off-equilibrium inaction selects a unique equilibrium. And there is no way to tell apart (56) from (57) using data from a given equilibrium.

Specification (57) is not unreasonable. A government following (57) pays off debts it incurs, for example, to finance wars or fight recessions. If the government wants to borrow real resources and keep a stable price level P^*, then such commitment is vital. If the government wants inflation, it needs to commit *not* to pay off debts due to a rising price level. But a government could and arguably should refuse to accommodate bubbles and sunspots in the price level.

Empirically, governments do not always rally surpluses to unexpected inflations and deflations! For example, the US in the 1930s went off the gold standard and defaulted on the gold clause in its bonds after a sharp deflation. Governments can well refuse to validate price levels they don't like, whether "on" or "off" equilibria.

15.3 What about Japan? And Europe? And the US?

Japan's debt approaches 200% of GDP, and 20 years of deficit spending have not produced desired inflation. Doesn't the fiscal theory predict hyperinflation for Japan? Europe is north of 100% debt to GDP ratio, and the US fast approaching. Present value of surpluses? What surpluses? The CBO's deficit forecasts have even primary deficits exploding. How can one begin to write about fiscal policy anchoring inflation expectations?

If only the fiscal theory were as simple as deficits and debt equals inflation! It is not.

Despite awful projections, it is not unreasonable for bond markets to believe – for now – that the western world's debt problems will be solved successfully. The CBO's deficit forecasts are "if something doesn't change" forecasts, and include straightforward policies that can turn surpluses around – mild pro-growth economic policies, mild entitlement reforms. The same angst over debt-fueled inflation emerged in the late 1970s and early 1980s – Sargent and Wallace's "unpleasant monetarist arithmetic" being only the most famous example. (We remember the pathbreaking analysis of fiscal-monetary interactions. We politely forget the forecast that the US would soon return to inflation in the 1980s.) A few regulatory and tax reforms later, we were worrying in the late 1990s about the disappearance of government debt. The US largely repaid greater debts after WWII, and the UK grew out of much larger debt in the 1800s. A debt crisis leading to inflation will be a self-inflicted wound, not an economic necessity.

Much more importantly, in my view, the fiscal theory needs to digest the main lesson of asset pricing – discount rates vary a lot, and are vitally important to understand valuations.

Real interest rates are zero or negative throughout the western world, even at very long horizons. From this perspective, the fiscal theory puzzle is not the lack of *inflation.*

The fiscal theory puzzle is the lack of *deflation*.

For a simplified approach to this observation, consider a constant discount rate r and an economy growing at rate g – and hence primary surpluses $s_t = \tau y_t$ growing at g.

$$\frac{B_t}{P_t} = E_t \int_{j=0}^{\infty} e^{-rj} s_{t+j} dj = E_t \int_{j=0}^{\infty} e^{(g-r)j} dj s_t = \frac{s_t}{r-g} \tag{58}$$

or, the ratio of surplus to real value of the debt is $r - g$. The discount rate r matters as much as g. If r declines, the value of the debt rises, so P declines.

So, to understand low current inflation, the salient fact is the extraordinarily low expected real return on government debt. It is valuable despite poor fiscal prospects, not because of great ones.

15.4 What about 2008? And cyclical inflation?

Discount rate variation is likely also to be crucial to understanding episodes such as 2008, and the cyclical correlation of inflation.

Fall 2008. Output falls sharply. Deficits expand into the trillions. The growth slowdown and continuing entitlement problems make future deficits seem even worse. And inflation... plummets. How is that consistent with the government debt valuation equation? In every recession, low inflation correlates with large deficits, and vice versa. Isn't the sign wrong?

Again, it is a mistake to confuse current with expected future deficits. A government fighting a war borrows today and simultaneously commits to higher future taxes to pay off the debt – precisely because it does not want to create too much inflation. Our government's stimulus programs always include at least lip service to future deficit reduction. The surplus should *not* be modeled as an AR(1), but as a series in which a dip today portends a rise in the future. (Cochrane (2001) pursues this point in detail.) Contrariwise, "helicopter drop" plans to deliberately create inflation come with commitments not to repay the debt.

But future surpluses didn't get a lot *better* in 2008. The real answer is the discount rate. Both nominal and real interest rates dropped sharply in 2008. A "flight to quality" further lowered the expected return of government bonds relative to corporate bonds. People were trying to hold more government bonds – and to hold less private assets and to spend less to get government bonds.

For a bit more analytical perspective, we can write the government debt valuation formula in a form first introduced in (11)

$$\frac{B_{t-1}}{P_t} = E_t \sum_{j=0}^{\infty} \frac{1}{R_{t,t+j}} s_{t+j} \tag{59}$$

We can always discount any asset by its ex-post return. So, a lower expected return on government debt, coming either from a lower real risk free rate or from a lower (or more negative!) risk premium raises the value of government debt, and pushes the price level down.

This mechanism helps as well to understand the general cyclical correlation of inflation. In any recession, output falls, deficits rise, and yet inflation falls. Why? In part, people understand that current deficits correspond to larger future surpluses. But the most important part of the effect is likely that people are willing to hold claims to the same surplus at much lower rates of return in a recession than they are in a boom.

Why in a recession do people want to defer consumption, sending interest rates down, and why do they become more risk averse, sending government bond returns down relative to private assets? Why are real rates of return on government debt so low? Though good ideas for answers are plentiful, that is a cause beyond the current point: discount rate variation is plausible, and discount rates rather than expected surpluses likely account for the otherwise puzzling correlation or lack of correlation between debt, deficits and inflation.

Once again, the government debt valuation formula holds in the equilibrium of any model, "active" or "passive." Using ex-post returns to discount, it is an identity, which always holds. It is not a testable equation. So the issue, whether expected surplus variation or discount rate variation accounts for movements in the price level over time, is not particular to the fiscal theory. In a passive regime, getting this wrong is less important to other parts of the model, but the analysis does not rest on active vs. passive sources of price level and surplus variation.

15.5 What about 1951?

And what about all the other failed interest rate pegs? Answer: Fiscal policy.

In the face of inflationary fiscal policy, interest rate pegs cannot hold back inflation. For example, Woodford (2001) analyzes the US peg of the 1940s and early 1950s. He credits fiscal-theoretic mechanisms for the surprising stability of the interest rate peg – it did last a decade. In his view, it fell apart when the Korean war undermined fiscal policy. Other countries whose pegs fell apart after WWII motivating Friedman (1968) (quotes below), were facing difficult fiscal problems. Most historic pegs were enacted along with price controls and monetary controls as devices to reduce interest payments on the debt (that was explicit in the US case) and to help otherwise difficult fiscal policy. In most historic pegs, central banks were trying to hold down rates that otherwise wanted to rise, by lending out money to banks at low rates, and with financial repression to force people to hold government debt they did not want to hold. Our central banks are taking *in* money from banks who can't find better opportunities elsewhere, thus apparently holding interest rates above what they otherwise want to be, in the face of overwhelming demand for government debt. Countries whose pegs fell apart had problems financing current deficits. We have doomsaying forecasts of deficits decades from now. The lessons of historical pegs under vastly different fiscal circumstances do not necessarily apply to all pegs.

Interest rate pegs, like exchange rate pegs, live on top of solvent fiscal policy or not at all. For this reason I have been careful to state the doctrine as "an interest rate peg *can* be stable."

To be sure, in none of these cases – What about secular and cyclical low inflation with high debts, what about past pegs that blew up, etc. – do I attempt to provide a solid answer. The point here is not to establish these speculations as definitive answers. The point is that *there exists* a vaguely plausible reconciliation of the data with the government debt valuation equation; that the model passes the laugh test and cannot be instantly dismissed with obvious facts.

15.6 The uncomfortable future

So far, this paper has presented an unfashionably comfortable view of our future. Low interest rates just drag inflation down to the Friedman rule. Blips in inflation will melt away on their own accord under an interest rate peg. Neither instability nor indeterminacy need worry us. The huge balance sheet will just sit there. If the Fed were to raise rates, inflation will gently rise to meet the new higher rates, after a possible short small inflation decline buried deep in fiscal theory mechanics of long term debt.

But the tenuous fiscal underpinnings of our "anchoring" could fall apart quickly. Most debt is short term, rolled over much more quickly than the maturity of any plausible return to surpluses. Government debt is valued because each investor thinks he or she will be paid from the proceeds of a roll over. Short term debt and long term illiquid assets (the present value of a government's taxing power) are prone to runs.

As the factors driving low rates of return on government debt, low $r - g$ and hence high valuations for government debt, are a bit mysterious, that gives us little confidence against their sudden evaporation. Higher real interest rates times a large outstanding stock of debt will put a sharp dent into budgets. The debt sustainability literature often takes a view of multiple equilibria – low interest rates make debt sustainable, and result in low interest rates. Higher interest rates make debt unsustainable and lead to higher interest rates, via either default or inflation premiums. (Had our governments chosen much longer-term financing, we would not be exposed to runs or bad news in this way.)

The simple Gordon growth formula for the fiscal theory, (58) offers some insight to these questions. We are deep in the region where r is close to g, so small changes in r can have large effects.

On the other hand, small increases in our desultory growth rates g can have equally large disinflationary effects. Long-run GDP growth rates are the prime determinant of long-run surpluses.

So, there is both good and bad r. "Good r" is a rise in interest rates that results from a rise in the economic growth rate. From the basic first order condition,

$$r = \delta + \gamma(g - n)$$

where δ is the rate of time preference, $\gamma = 1/\sigma$ is the inverse substitution elasticity, and n is population growth so $g-n$ is per capita growth. If $\gamma = 1$, log utility, then an increase in economic growth will increase real interest rates one for one, and we live in a happy world in which the value of government debt is unaffected by growth or interest rate variation. Typical calibrations put γ slightly above one, but primary surpluses also are more than 1-1 sensitive to economic growth, especially when deficits are so strongly the result of entitlement programs rather than discretionary spending.

"Bad r" is a rise in expected rate of return on government debt that comes without a rise in economic growth, or with a decline in economic growth. If bond markets get scared of a default, or inflation, the risk premium on government debt will rise, and we get the full inflationary effect of a rising rate of return with no concurrent growth rate effects.

The bottom line: some historic interest rate pegs, like exchange rate pegs and the gold standard, lasted a surprisingly long time. Many interest rate pegs fell apart when their fiscal foundations fell apart. With short term debt, that can happen in what feels like "speculative attack" "bubble" or "run" to central bankers. Inflation's resurgence can happen without Phillips curve tightness, and can surprise central bankers of the 2020s just as it did in the 1970s, and just as inflation's decline surprised them in the 1980s, and its stability surprised them in the 2010s.

16 Literature

I boil these points down to very simple models, but we should not lose sight of how deep and widespread the doctrines of inflation instability or indeterminacy are, how radical it is to suggest that those doctrines are mistaken.

Milton Friedman (1968) gives the classic statement that an interest rate peg is unstable, and that higher interest rates lead to temporarily lower inflation. Friedman writes (p.5) that Monetary policy "cannot peg interest rates for more than very limited periods..." Friedman's prediction also comes clearly from adaptive expectations: (p. 5-6):

> "Let the higher rate of monetary growth produce rising prices, and let

> the public come to expect that prices will continue to rise. Borrowers will then be willing to pay and lenders will then demand higher interest rates-as Irving Fisher pointed out decades ago. This price expectation effect is slow to develop and also slow to disappear."

Friedman stressed a money growth channel rather than an IS channel – low interest rates require money growth; money growth leads to more spending and eventually more inflation, more inflation puts upward pressure on interest rates unless money growth increased further, and so on. But the bottomline dynamics from interest rate to inflation does not depend on this view of the mechanism. The very simple Keynesian model of Figure 4 captures completely Friedman's description of inflation instability and interest rate policy under adaptive expectations.

Friedman was heavily influenced by recent history of his time:

> These views [ineffectiveness of monetary policies] produced a widespread adoption of cheap money policies after the war. And they received a rude shock when these policies failed in country after country, when central bank after central bank was forced to give up the pretense that it could indefinitely keep "the" rate of interest at a low level. In this country, the public denouement came with the Federal Reserve-Treasury Accord in 1951, although the policy of pegging government bond prices was not formally abandoned until 1953. Inflation, stimulated by cheap money policies, not the widely heralded postwar depression, turned out to be the order of the day. The result was the beginning of a revival of belief in the potency of monetary policy.

Above I noted that all countries including the US, were also managing fiscal policy stresses and large wartime debts, and that the bond-pegging policy worked for a surprisingly long time for an unstable equilibrium.

Most of all, we do not know how Friedman, an intensely empirical economist, might have adapted his views on the stability of an interest rate peg with our recent 8 years of experience, or Japan's 20 in mind, rather than the inflations of the immediate WWII aftermath.

Though the literature developed over 20 years, Taylor (1999) provides a clear statement that old-Keynesian (backward-looking, adaptive expectations) models are unstable, and that Taylor rules induce stability.

The doctrine that inflation is *indeterminate* under an interest rate peg or passive policy, under rational expectations, started with Sargent and Wallace (1975). The basic point: fixing i_t with $i_t = r_t + E_t\pi_{t+1}$ leaves $\pi_{t+1} - E_t\pi_{t+1}$ indeterminate. Though often confused, their point is quite different from Friedman's. Indeterminacy is not the same thing as instability, and neither is the same thing as volatility!

The fiscal theory of the price level goes back to Adam Smith:

> "A prince who should enact that a certain proportion of his taxes should be paid in a paper money of a certain kind might thereby give a certain value to this paper money" – Wealth of Nations, Book 2, Ch. II.

Monetary economists have always recognized the importance of monetary-fiscal interactions. It did not escape notice that inflations since Diocletian's stem from fiscal problems, and that printing up money to cover debts as in our revolutionary and civil wars causes inflation. Much of the structure of our monetary arrangements comes down to institutions that prevent inflationary finance.

The modern resurgence and deep elaboration owes much to Sargent (2013) (first published in the early 1980s). Leeper (1991) showed how the fiscal theory can uniquely determine the price level under passive monetary policy. Sims (1994) clearly states that the fiscal theory and rational expectations overturn Friedman's doctrine of instability, as well as Sargent and Wallace's indeterminacy:

> "A monetary policy that fixes the nominal interest rate, even if it holds the interest rate constant regardless of the observed rate of inflation or money growth rate, may deliver a uniquely determined price level."

The observation that interest rate pegs are stable in forward-looking new Keynesian models also goes back a long way. Woodford (1995) discusses the issue. Woodford (2001) is a clear statement, analyzing interest rate pegs such as the WWII US price support regime, showing they are stable so long as fiscal policy cooperates.

Benhabib, Schmitt-Grohé, and Uribe (2002) is a classic treatment of the zero-rate liquidity trap. They note that the zero bound means there must be an equilibrium with a locally passive $\phi_\pi < 1$ Taylor rule, with multiple stable equilibria. However, they view this state as a pathology, not a realization of the optimal quantity of money and optimal (low) level of markups, and devote the main point of their paper to escaping the "trap" via fiscal policy.

Schmitt-Grohé and Uribe (2014) realize that inflation stability means the Fed could raise the peg and therefore raise inflation. To them this is another possibility for escaping a liquidity trap. They write

> "The paper... shows that raising the nominal interest rate to its intended target for an extended period of time, rather than exacerbating the recession as conventional wisdom would have it, can boost inflationary expectations and thereby foster employment."

The simple model here disagrees that raising inflation raises employment. With a forward-looking Phillips curve, rising inflation reduces employment here, as in Figure 11.

Belaygorod and Dueker (2009) and Castelnuovo and Surico (2010) estimate new-Keynesian / DSGE models allowing for switches between determinacy and indeterminacy. They find that the model displays the price puzzle – interest rate shocks lead to rising inflation, starting immediately – in the indeterminacy region $\phi_\pi < 1$, as I do.

The possibility long periods of low rates cause deflation, so raising interest rates will raise inflation, has had a larger recent airing in speeches and the blogosphere. (Williamson (2013), Cochrane (2013, 2014b), Smith (2014).) Minneapolis Federal Reserve Chairman Narayana Kocherlakota (2010) suggested it in a famous speech:

> "Long-run monetary neutrality is an uncontroversial, simple, but nonetheless profound proposition. In particular, it implies that if the FOMC maintains the fed funds rate at its current level of 0-25 basis points for too long, both anticipated and actual inflation have to become negative. Why? It's simple arithmetic. Let's say that the real rate of return on safe investments is 1 percent and we need to add an amount of anticipated inflation that will

> result in a fed funds rate of 0.25 percent. The only way to get that is to add a negative number – in this case, 0.75 percent.
>
> To sum up, over the long run, a low fed funds rate must lead to consistent– but low–levels of deflation."

To be clear, Friedman (1968) disagrees. Friedman views the Fisher equation as a steady state, but it is an unstable steady state. Kocherlakota, seeing recent experience, views it as a stable one. Friedman's "long-run neutrality" is that a permanent rise in M gives rise to a permanent rise in P, but a permanent rise in i will lead to explosive P and π. A permanent rise in $\dot{M}$ gives rise to a permenent rise in $\dot{P}$ and therefore of both i and π, but not the other way around . Kocherlakota's definition of "long-run neutrality" is that a permanent rise in i will lead to a permanent rise in π. The difference between a stable and an unstable steady state is key.

Cochrane (2014a) works out a model with fiscal price determination, an interest rate target, and simple k-period price stickiness. Higher interest rates raise inflation in the short and long run, just as in this paper, but the k-period stickiness leads to unrealistic dynamics.

Following Woodford (2003), many authors have also tried putting money back into sticky-price models. Benchimol and Fourçans (2012) and Benchimol and Fourçans (2015) study a CES money in the utility function specification as here, in a detailed model applied to the Eurozone. They find that adding money makes small but important differences to the estimated model dynamics.

Ireland (2004) also adds money in the utility function. In his model, money also spills over into the Phillips curve. He writes, (p. 974)

> "... optimizing firms set prices on the basis of marginal costs; hence, the measure of real economic activity that belongs in a forward-looking Phillips curve ... is a measure of real marginal costs, rather than a measure of de-trended output ... in this model, real marginal costs depend on real wages, which are in turn linked to the optimizing household's marginal rate of substitution between consumption and leisure. Once again, when utility is non-separable, real balances affect this marginal rate of substitution; hence, in

this case, they also appear in the Phillips curve."

However, he finds that maximum likelihood estimates lead to very small influences of money, a very small if not zero cross partial derivative u_{cm}.

Where Ireland's Phillips curve comes from quadratic adjustment costs, Andrés, López-Salido, and Vallés (2006) find a similar result from a Calvo-style pricing model. Their estimate also finds no effects of money on model dynamics.

Many have noted that the stability and quiet (lack of volatility) of inflation at steady interest rates is a puzzle. Homburg (2015) points out Japan's "benign liquidity trap" and the deep puzzle it poses for standard models. He advocates a repair with pervasive credit constraints.

17 Concluding comments

The observation that inflation has been stable or gently declining and quiet at the zero is important evidence against the classical view that an interest rate peg must be unstable, and the new-Keynesian view that it leads to sunspot volatility. If an interest rate peg is stable, then raising the interest rate peg should raise inflation, at least eventually.

Conventional new-Keynesian models predict that inflation is stable. Adding the fiscal theory of the price level, or related rules for selecting nearby equilibria, removes indeterminacies and sunspots and leads to a very simple monetary model consistent with our recent experience

Those models also predict that raising interest rates will raise inflation, both in the long and short run. My attempts to escape this prediction by adding money, backward looking Phillips curves, multiple equilibria or Taylor rules all fail.

The new-Keynesian model plus fiscal theory and long-term debt does produce a temporary negative inflation response to unexpected interest rate increases. It is "simple," and "economic," but quite novel relative to standard monetary models. It does not produce the standard, adaptive-expectations view of a permanent disinflation such as the 1980s, nor does it justify policy exploitation of the negative sign, especially in systematic, Taylor-principle form.

This paper was also a search for a simple model that captures the effects of monetary policy, but overcomes the critiques of active and instability-inducing Taylor rules in forward-looking models. The new-Keynesian plus fiscal theory model in this paper satisfies that criterion.

A review of the empirical evidence finds very weak support for the standard theoretical view that raising interest rates lowers inflation, and much of that evidence is colored by the imposition of strong priors of that sign.

I conclude that a positive reaction of inflation to interest rate changes is a possibility we, and central bankers, ought to begin to take seriously.

Both the fact of quiet inflation and the theory here rehabilitate interest rate pegs as at least a possible policy. We can live the Friedman rule of low interest, low inflation, and enormous reserves. Real policies may choose a time-varying peg if central banks think they can offset shocks (v_t^i here may react to v_t^r), and may desire headroom of higher inflation to do that. But there is no need to fear catastrophe of inflation from the former policy configuration.

However, that quiet depends on fiscal foundations. The large demand for government debt which provides the fiscal foundations of this quiet (under either "active" or "passive" views), driven by low discount rates not strong fiscal policies, could evaporate as unexpectedly as it arrived.

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18 Appendix. A Phillips curve with lags

The usual Phillips curve (19) is forward looking:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t \tag{60}$$

or equivalently

$$\pi_t = \kappa \sum_{j=0}^{\infty} \beta^j E_t x_{t+j}. \tag{61}$$

To incorporate lagged inflation, I write instead

$$\pi_t = \kappa \left(x_t + \sum_{j=1}^{\infty} \phi^j E_t x_{t+j} + \sum_{j=1}^{\infty} \rho^j x_{t-j} \right) \tag{62}$$

or, in autoregressive form,

$$\pi_t = \frac{\phi}{1+\rho\phi} E_t \pi_{t+1} + \frac{\rho}{1+\rho\phi} \pi_{t-1} + \frac{(1-\phi\rho)}{(1+\phi\rho)} \kappa x_t.$$

So that the sum of coefficients on the right hand side of (62) is the same as it is in (61), and hence so that the steady state relationship between output and inflation remains unchanged, I constrain the backward and forward looking coefficients ρ and ϕ to satisfy

$$\frac{(1-\phi)(1-\rho)}{(1-\phi\rho)} = (1-\beta)\,. \tag{63}$$

Repeating the model solution, inflation is again a two-sided moving average of interest rates, and in the presence of money of expected changes in interest rates,

$$\pi_{t+1} = \kappa\sigma \frac{(1-\phi\rho)}{\phi} \frac{\lambda_3}{\left(1-\lambda_3\lambda_1^{-1}\right)\left(1-\lambda_3\lambda_2^{-1}\right)} \times \left(E_t z_t + \sum_{j=1}^{\infty} \lambda_3^j E_t z_{t+j} + \right.$$

$$\left. + \frac{\lambda_1^{-1}\left(1-\lambda_2^{-1}\lambda_3\right)}{\left(\lambda_1^{-1}-\lambda_2^{-1}\right)} \sum_{j=1}^{\infty} \lambda_1^{-j} E_{t-j} z_{t-j} - \frac{\lambda_2^{-1}\left(1-\lambda_1^{-1}\lambda_3\right)}{\left(\lambda_1^{-1}-\lambda_2^{-1}\right)} \sum_{j=0}^{\infty} \lambda_2^{-j} E_{t-j} z_{t-j} \right) \tag{64}$$

where $\lambda_1, \lambda_2, \lambda_3$ are the roots of the polynomial in

$$-\frac{\rho}{\phi}+\frac{(1+\rho(1+\phi))}{\phi}L^{-1}-\frac{(1+\phi+\kappa\sigma+\phi\rho(1-\kappa\sigma))}{\phi}L^{-2}+L^{-3}$$

$$=\left(L^{-1}-\lambda_1^{-1}\right)\left(L^{-1}-\lambda_2^{-1}\right)\left(L^{-1}-\lambda_3^{-1}\right)$$

The expression (64) now has two backward looking moving averages as well as one forward looking term. The long-run response of inflation to interest rates remains one.

18.1 Roots, moving average, and response

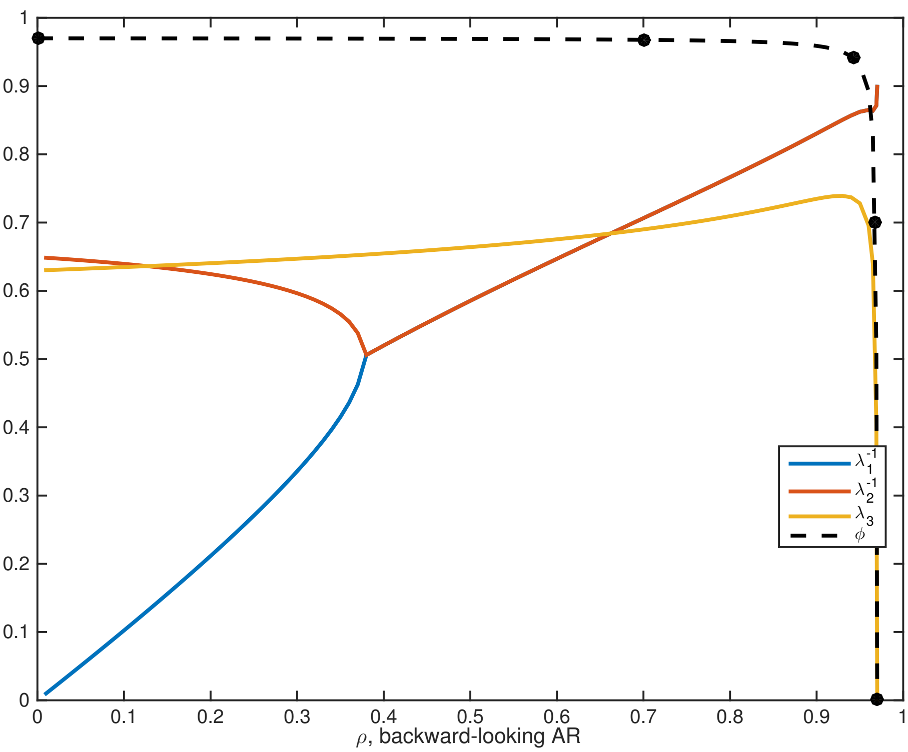

Figure 28: Roots λ of the impulse response function, and forward looking Phillips curve parameter ϕ, for each choice of the backward looking Phillips curve parameter ρ.

Figure 28 displays for each value of the backward looking weight ρ in the Phillips curve with lagged inflation (62), the forward looking weight ϕ given by restriction (63), and the three roots $\lambda_1^{-1}, \lambda_2^{-1}$ and λ_3 that govern the moving averages (64).

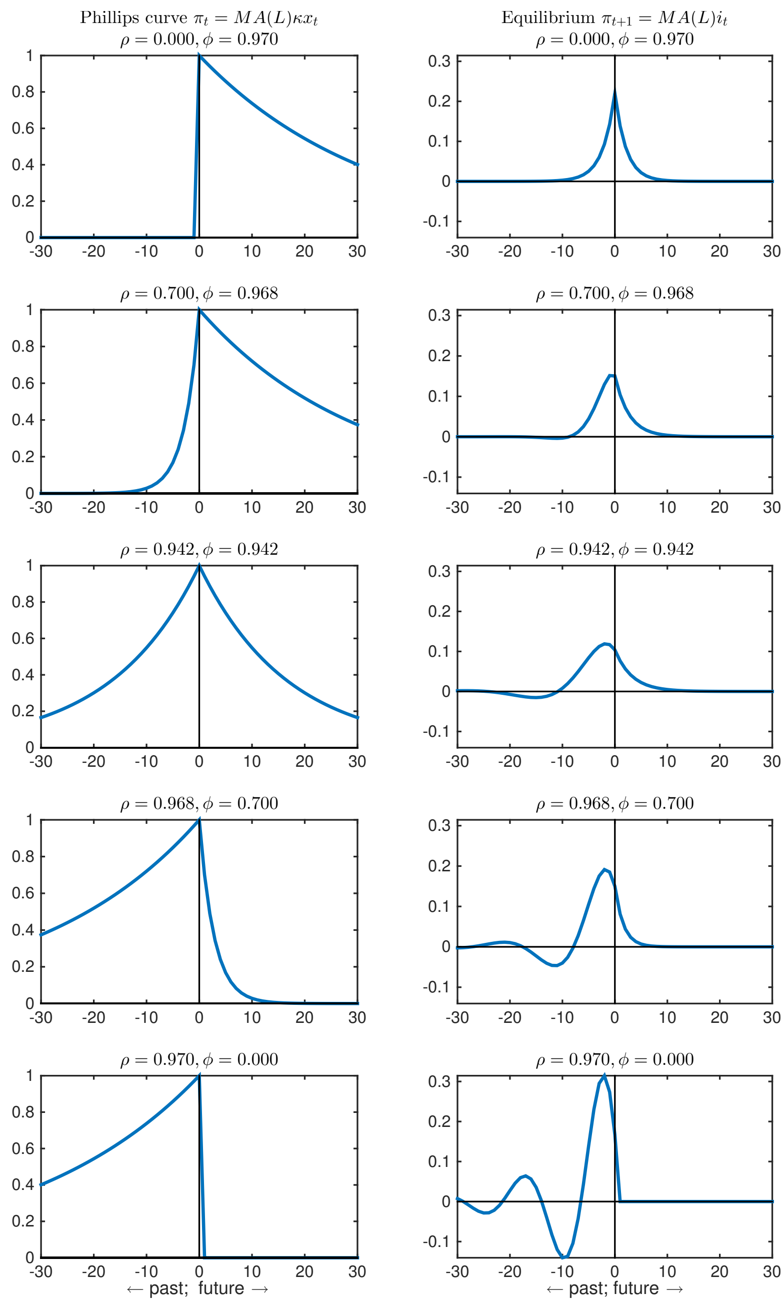

Figure 29: Moving average representation of the two-sided Phillips curve, and corresponding moving-average response of inflation to interest rates.

The left hand column of Figure 29 presents the moving average representation of the Phillips curve (62), and the right hand column presents the moving average coefficients of the solution (64), for specific choices of ρ and the consequent ϕ. These specific values of $\rho,\ \phi$ are represented as dark circles in Figure 28.

Starting at the left of Figure 28, and the top of Figure 29, we have the previous case $\rho = 0,\ \phi = \beta$ of a purely forward looking Phillips curve. Figure 28 shows the nearly equal forward and backward looking roots λ_2^{-1} and λ_3, and $\phi = \beta$. The top left element of Figure 29 shows the purely forward looking weights of the Phillips curve, while the top right element shows the nearly equal forward and backward moving average weights of the model's solution for inflation as a function of interest rates.

As we raise the backward looking coefficient ρ, Figure 28 shows that the forward looking coefficient ϕ, the forward looking root in the solution λ_3 and the original backward looking root λ_2^{-1} change little. We bring in a second backward looking root, roughly equal to ρ itself. Around $\rho = 0.55$, the two backward looking roots become complex. Their magnitude is still less than one, but the complex nature will generate a damped sinusoidal response function.

The second row of Figure 29 shows the Phillips weights and response function for a backward Phillips curve coefficient $\rho = 0.7$. The forward looking coefficient $\phi \approx \beta$ is still large, so this case captures a small amount of backward looking behavior, and helps us to assess if a small amount of such behavior can substantially change results. The moving average solution in the right column is still basically two-sided and positive. It begins to weight the past more than the future. The small change in this moving average previews the result below that this small amount of backward looking behavior in the Phillips curve will not materially affect the neo-Fisherian response to an interest rate rise.

Continuing to the right in Figure 28, the backward looking roots continue to grow nearly linearly with the backward looking Phillips parameter ρ. The forward looking coefficient ϕ and the forward looking root λ_3 remain nearly unaffected however, for even very large values of ρ.

The third row of Figure 29 shows the case $\rho = \phi$ that the Phillips curve is equally backward- and forward looking. The right column shows that the response function is now weighted more to the past than the future. In addition, negative coefficients are starting to show up, giving us some hope that higher interest rates can result in lower inflation at some point along the dynamic path.

The fourth and fifth rows of Figure 29 show cases in which the Phillips curve becomes more and more backward looking. The fourth row shows a forward weight reduced to $\phi = 0.7$, and the fifth row shows the purely backward looking case $\rho = 0$, $\phi = \beta$. Figure 28 shows that only for very large values of the backward looking coefficient ρ near $\rho = \beta$ do ϕ and the forward looking root λ_3 substantially decline. At that limit, both forward looking terms disappear, and the two complex backward looking roots remain. Figure 29 shows that the moving average solution becomes more and more weighted to past values, with larger sinusoidal movements.

Figure 30 presents the response of inflation, on the left, and output, on the right, to the standard step function interest rate path, for the same choices of forward and backward looking Phillips curve parameters as in the last two figures. The dashed line in each case is the unexpected case, verifying that once again expected and unexpected paths are the same for dates after the announcement.

Starting at the top of Figure 30, we have the purely forward looking case $\rho = 0,\ \phi = \beta$, and the same result as before. Inflation rises smoothly to meet the higher interest rate, and the Phillips curve produces a small output reduction on the whole path.

In the second row, a little bit of backward looking behavior $\rho = 0.7$ produces a plot that is almost visually indistinguishable. Inflation rises throughout, and output is still depressed until the long-run inflation-output tradeoff of this model takes hold. So, the basic result is robust to adding backward looking behavior.

As we go to more and more backward looking behavior in the remaining rows of Figure 30, inflation and output cease to respond ahead of the funds rate rise. Backward looking Phillips curves mean that forward guidance has less and less effect.

However, in none of the cases does a rise in interest rates provoke a decline in inflation. We can see the reason in the moving average coefficients of Figure 29. Though that figure does have some negative coefficients, in which past interest rates lower current inflation, the coefficients at low lags are always positive, and outweigh the negative

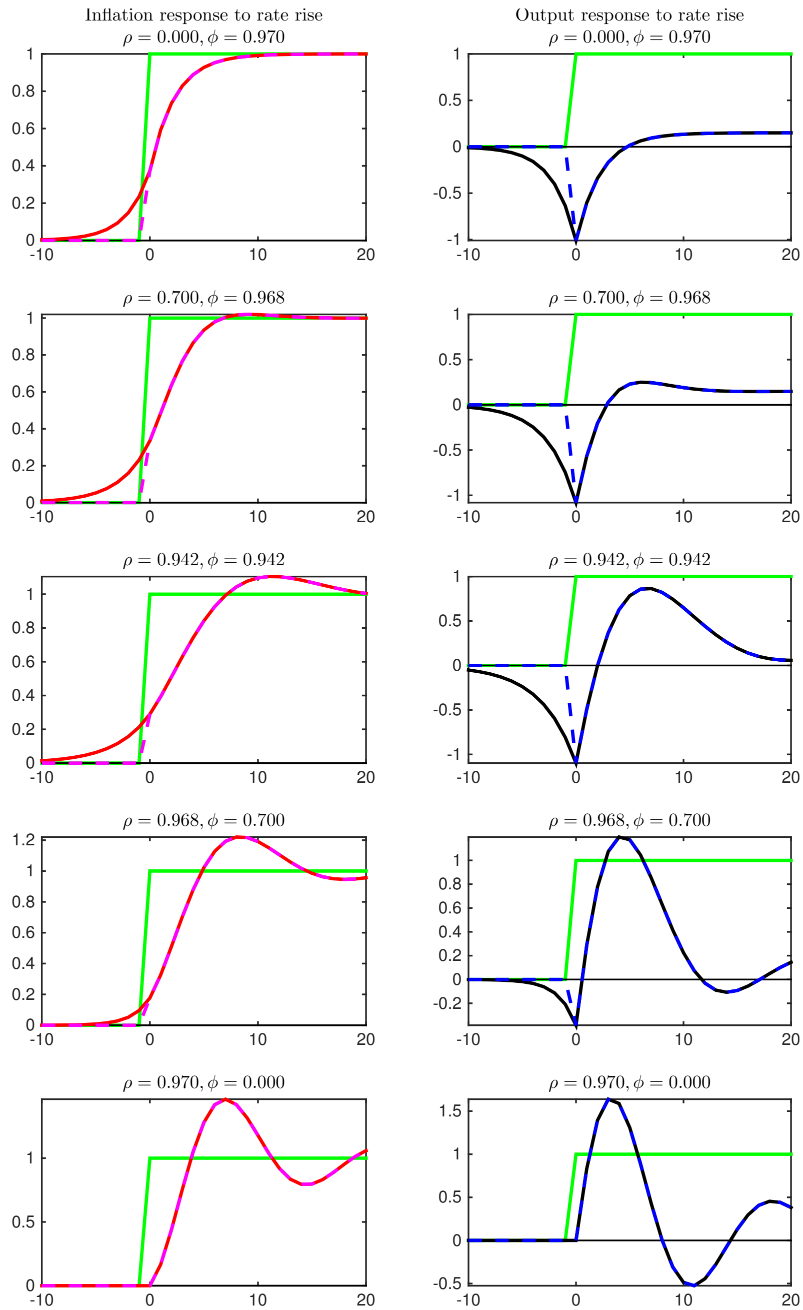

Figure 30: Response of inflation and output to a step-function rise in interest rates, with lagged inflation in the Phillips curve. Solid lines are the response to an expected change, dashed lines are the response to an unexpected change. The backward-looking Phillips parameter is ρ, and ϕ is the forward-looking parameter.

coefficients further in the past. Integrating, we see the overshooting behavior in the bottom of Figure 30.

As the Phillips curve becomes more backward looking, the output decline with an interest rate rise weakens, and eventually becomes an output rise. While the forward looking Phillips curve gives higher output when inflation is declining, the backward looking Phillips curve gives higher output when inflation is rising. In this experiment inflation does rise, so output rises as well. Moving to a backward looking Phillips curve, we did turn around a sign: the "right" decline in output turned into a "wrong" rise in output, leaving the "wrong" rise in inflation alone.

Together, then, the backward looking Phillips curve and the neo-Fisherian behavior of inflation mean that in interest rate rise looks much like what is conventionally expected of a monetary *expansion*, not a contraction, plus some interesting slow sinusoidal dynamics.

Figure 31 adds both money and a purely backward looking Phillips curve. Compare this result to Figure 17 for money with a forward looking Phillips curve, and to the bottom row of Figure 30 for a backward looking Phillips curve without money.

Figure 31 produces something like the standard intuition, for inflation, at last. The unanticipated rate rise still does not interact with money at all, so it produces the same response for all values of money m/c. But the anticipated rate rise now benefits from the postponement of its response, from the backward looking Phillips curve, and also the reductions in near-term inflation from adding money. Now there is a temporary reduction in inflation before inflation rises to join the interest rate, for any positive value of money m/c.

Output now also falls, before rising with inflation. The backward looking Phillips curve generates low output when inflation is decreasing, and high output when inflation is increasing.

However, comparing the results to Figure 17 with a forward looking Phillips curve, the benefit is small. That figure already showed a temporary inflation decline, and no oscillating dynamics. The inflation decline is larger now, and smoother, but not dramatically different. We still need substantial monetary distortions $m/c > 1$ to obtain a

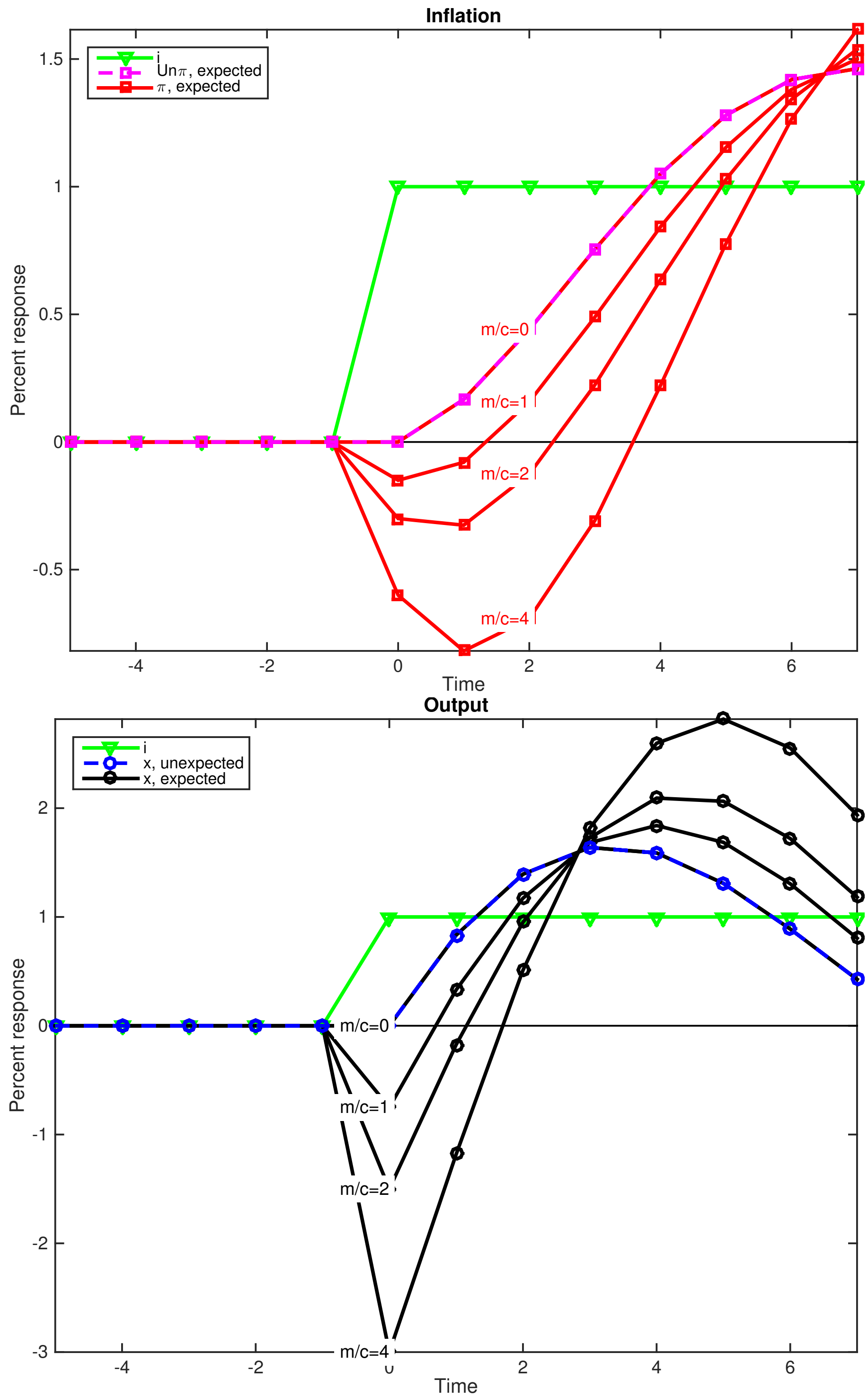

Figure 31: Response to expected and unexpected interest rate rise, with money and a purely backward looking Phillips curve $\rho = \beta$, $\phi = 0$.

quantitatively interesting response, or large $\sigma > 1$ (not shown).

The conventional sign of the short-run output response along with lower short-run inflation is perhaps the greatest benefit.

But the cost is throwing out all of the forward looking optimizing microfoundations of the forward looking Phillips curve. Anything much less that purely backward looking behavior (not shown) does not produce significant improvements.

Also, since unanticipated interest rate rises have no interaction with money, this modification does not help to match VAR evidence and intuition that focuses on unanticipated changes in interest rates.

Finally, the long-run responses still defy conventional intuition, losing the smooth decline present in the simple model of Figure 11. The disinflation and output cooling are borrowed from future inflation and an output boom.

18.2 Backward-looking Phillips algebra

The Phillips curve with lagged inflation is

$$\begin{aligned}\pi_t &= \kappa\left(x_t + E_t\sum_{j=1}^{\infty}\phi^j x_{t+j} + \sum_{j=1}^{\infty}\rho^j x_{t-j}\right)\\ &= E_t\left(1+\frac{\phi L^{-1}}{1-\phi L^{-1}}+\frac{\rho L}{1-\rho L}\right)\kappa x_t\end{aligned}$$

or, in autoregressive form

$$\pi_t = E_t\left(\frac{1-\rho\phi}{\left(1-\phi L^{-1}\right)\left(1-\rho L\right)}\right)\kappa x_t \tag{65}$$

$$E_t\left(1+\rho\phi-\phi L^{-1}-\rho L\right)\pi_t = \left(1-\rho\phi\right)\kappa x_t.$$

$$\pi_t = \frac{\phi}{1+\rho\phi}E_t\pi_{t+1} + \frac{\rho}{1+\rho\phi}\pi_{t-1} + \frac{(1-\phi\rho)}{(1+\phi\rho)}\kappa x_t.$$

To maintain the same steady state relationship between output and inflation, I constrain ρ and ϕ to satisfy

$$\frac{(1-\phi\rho)}{(1-\phi)(1-\rho)} = \frac{1}{(1-\beta)}.$$

So, for each choice of the weight ρ on past inflation, I use a weight ϕ on forward inflation given by

$$\phi = \frac{\beta - \rho}{1 + \beta\rho - 2\rho}. \tag{66}$$

The case $\rho = \phi$ occurs where

$$\rho = \frac{\beta}{2 - \beta}.$$

Now, to solve the model. To keep the algebra simple I find the perfect foresight solution and put the E_t back in at the end. The IS curve is

$$x_t = x_{t+1} + \sigma\pi_{t+1} - \sigma z_t$$

$$(1 - L^{-1})x_t = \sigma\left(L^{-1}\pi_t - z_t\right)$$

Forward-differencing (65) and substituting,

$$(1 - L^{-1})\pi_t = \left(\frac{1 - \phi\rho}{(1 - \phi L^{-1})(1 - \rho L)}\right)\left(1 - L^{-1}\right)\kappa x_t$$

$$(1 - L^{-1})\pi_t = \kappa\sigma\left(\frac{1 - \phi\rho}{(1 - \phi L^{-1})(1 - \rho L)}\right)(L^{-1}\pi_t - z_t)$$

$$\left[(1 - L^{-1})\left(1 - \phi L^{-1}\right)(1 - \rho L) - \kappa\sigma(1 - \phi\rho)L^{-1}\right]\pi_t = -\kappa\sigma(1 - \phi\rho)z_t$$

$$\left[-\rho L + (1 + \rho(1 + \phi)) - (1 + \phi + \kappa\sigma + \phi\rho(1 - \kappa\sigma))L^{-1} + \phi L^{-2}\right]\pi_t = -\kappa\sigma(1 - \phi\rho)z_t$$

$$\left[-\frac{\rho}{\phi} + \frac{1 + \rho(1 + \phi)}{\phi}L^{-1} - \frac{1 + \phi + \kappa\sigma + \phi\rho(1 - \kappa\sigma)}{\phi}L^{-2} + L^{-3}\right]\phi L\pi_t = -\kappa\sigma(1 - \phi\rho)z_t$$

Denoting the three roots of the lag polynomial λ_i^{-1},

$$E_t\left(L^{-1} - \lambda_1^{-1}\right)\left(L^{-1} - \lambda_2^{-1}\right)\left(L^{-1} - \lambda_3^{-1}\right)\phi L\pi_t = -\kappa\sigma(1 - \phi\rho)z_t.$$

I find these roots numerically. Nonetheless we can characterize them somewhat. Evaluating the lag polynomial at $L = 0$ we have

$$\lambda_1\lambda_2\lambda_3 = \frac{\phi}{\rho}. \tag{67}$$

while $L=1$ gives

$$\left(1-\lambda_1^{-1}\right)\left(1-\lambda_2^{-1}\right)\left(1-\lambda_3^{-1}\right)=\left(-\frac{\rho}{\phi}+\frac{1+\rho\,(1+\phi)}{\phi}-\frac{1+\phi+\kappa\sigma+\phi\rho\,(1-\kappa\sigma)}{\phi}+1\right)$$

$$\left(1-\lambda_1^{-1}\right)\left(1-\lambda_2^{-1}\right)\left(1-\lambda_3^{-1}\right)=-\sigma\kappa\frac{(1-\phi\rho)}{\phi}$$

We will have $\lambda_1,\lambda_2>1$, $\lambda_3<1$ so it's convenient to write the result as

$$\lambda_3^{-1}\left[\left(1-\lambda_1^{-1}L\right)\left(1-\lambda_2^{-1}L\right)\left(1-\lambda_3L^{-1}\right)\right]\phi L^{-1}\pi_t=\kappa\sigma\,(1-\phi\rho)\,z_t$$

$$\pi_{t+1}=\lambda_3\frac{\kappa\sigma}{\phi}\,(1-\phi\rho)\,\frac{1}{\left(1-\lambda_1^{-1}L\right)\left(1-\lambda_2^{-1}L\right)\left(1-\lambda_3L^{-1}\right)}z_t$$

I use the partial fractions identity

$$\frac{\lambda_3}{\left(1-\lambda_1^{-1}L\right)\left(1-\lambda_2^{-1}L\right)\left(1-\lambda_3L^{-1}\right)}==\frac{\lambda_3}{\left(1-\lambda_3\lambda_1^{-1}\right)\left(1-\lambda_3\lambda_2^{-1}\right)}\times$$

$$\times\left(1+\frac{\lambda_3L^{-1}}{\left(1-\lambda_3L^{-1}\right)}+\frac{\lambda_1^{-1}\left(1-\lambda_2^{-1}\lambda_3\right)}{\left(\lambda_1^{-1}-\lambda_2^{-1}\right)}\frac{\lambda_1^{-1}L}{1-\lambda_1^{-1}L}-\frac{\lambda_2^{-1}\left(1-\lambda_1^{-1}\lambda_3\right)}{\left(\lambda_1^{-1}-\lambda_2^{-1}\right)}\frac{\lambda_2^{-1}L}{1-\lambda_2^{-1}L}\right)$$

This takes pages of algebra to derive. It's easier just to check it. Thus, and reinserting the E_t

$$\pi_{t+1}=\kappa\sigma\frac{(1-\phi\rho)}{\phi}\frac{\lambda_3}{\left(1-\lambda_3\lambda_1^{-1}\right)\left(1-\lambda_3\lambda_2^{-1}\right)}\times$$

$$\times E_{t+1}\left[\left(1+\frac{\lambda_3L^{-1}}{\left(1-\lambda_3L^{-1}\right)}+\frac{\lambda_1^{-1}\left(1-\lambda_2^{-1}\lambda_3\right)}{\left(\lambda_1^{-1}-\lambda_2^{-1}\right)}\frac{\lambda_1^{-1}L}{1-\lambda_1^{-1}L}-\frac{\lambda_2^{-1}\left(1-\lambda_1^{-1}\lambda_3\right)}{\left(\lambda_1^{-1}-\lambda_2^{-1}\right)}\frac{\lambda_2^{-1}L}{1-\lambda_2^{-1}L}\right)E_tz_t\right] \tag{68}$$

The long-run response $(L=1)$ is

$$\begin{aligned}\pi_{t+1}&=\lambda_3\frac{\kappa\sigma}{\phi}\,(1-\phi\rho)\,\frac{1}{\left(1-\lambda_1^{-1}\right)\left(1-\lambda_2^{-1}\right)(1-\lambda_3)}E_tz_t\\&=-\frac{\kappa\sigma}{\phi}\,(1-\phi\rho)\,\frac{1}{\left(1-\lambda_1^{-1}\right)\left(1-\lambda_2^{-1}\right)\left(1-\lambda_3^{-1}\right)}E_tz_t\\&=\frac{\kappa\sigma}{\phi}\,(1-\phi\rho)\,\frac{1}{\sigma\kappa}\frac{\phi}{(1-\phi\rho)}E_tz_t\\&=1E_tz_t\end{aligned}$$

The case $\rho = 0, \phi = \beta$ is the conventional forward looking curve. The case $\phi = 0$, $\rho = \beta$, is a purely backward looking curve. In this case, the solution is

$$\left[\rho - (1+\rho) L^{-1} + (1+\kappa\sigma) L^{-2}\right] L\pi_t = \kappa\sigma z_t$$

$$\left(\frac{\rho}{(1+\kappa\sigma)} - \frac{(1+\rho)}{(1+\kappa\sigma)}L^{-1} + L^{-2}\right)(1+\kappa\sigma) L\pi_t = \kappa\sigma z_t$$

$$\left[\left(L^{-1}-\lambda_1^{-1}\right)\left(L^{-1}-\lambda_2^{-1}\right)\right](1+\kappa\sigma) L\pi_t = \kappa\sigma (1-\phi\rho) z_t$$

$$\lambda_1^{-1} = \frac{(1+\rho) \pm \sqrt{(1+\rho)^2 - 4\rho(1+\kappa\sigma)}}{2(1+\kappa\sigma)}$$

$$\lambda_2^{-1} = \frac{(1+\rho) \pm \sqrt{(1-\rho)^2 - 4\rho\kappa\sigma}}{2(1+\kappa\sigma)}$$

$$\lambda_3 = 0$$

$$\left[\left(1-\lambda_1^{-1}L\right)\left(1-\lambda_2^{-1}L\right)\right](1+\kappa\sigma) L^{-1}\pi_t = \kappa\sigma z_t$$

$$\pi_{t+1} = \frac{\kappa\sigma}{(1+\kappa\sigma)} \frac{1}{\left(1-\lambda_1^{-1}L\right)\left(1-\lambda_2^{-1}L\right)} z_t$$

$$\pi_{t+1} = \frac{\kappa\sigma}{(1+\kappa\sigma)}\left(1 + \frac{1}{\lambda_1^{-1}-\lambda_2^{-1}}\left(\lambda_1^{-1}\frac{\lambda_1^{-1}L}{\left(1-\lambda_1^{-1}L\right)} - \lambda_2^{-1}\frac{\lambda_2^{-1}L}{\left(1-\lambda_2^{-1}L\right)}\right)\right) z_t$$

19 Algebra Appendix

19.1 Variance of inflation in forward vs. backward looking models

This appendix works out the variance of inflation under the forward-looking vs. backward-looking solutions, equation (16).

The forward-looking solution is (9)

$$\pi_t = -\sum_{j=0}^{\infty}\left(\frac{1+\sigma\kappa}{1+\sigma\kappa\phi}\right)^{j+1}\frac{\sigma\kappa}{1+\sigma\kappa}E_t(v^i_{t+j} - v^r_{t+j}).$$

With $v^i = 0$ and $v_t^r = \rho v_{t-1}^r + \varepsilon_t^r$,

$$\pi_t = \sum_{j=0}^{\infty} \left(\frac{1+\sigma\kappa}{1+\sigma\kappa\phi} \right)^{j+1} \frac{\sigma\kappa}{1+\sigma\kappa} \rho^j v_t^r.$$

$$\pi_t = \left(\frac{\sigma\kappa}{1+\sigma\kappa\phi} \right) \frac{1}{1 - \left(\frac{1+\sigma\kappa}{1+\sigma\kappa\phi} \right) \rho} v_t^r.$$

$$\pi_t = \frac{\sigma\kappa}{1+\sigma\kappa\phi - (1+\sigma\kappa)\rho} v_t^r.$$

$$\pi_t = \frac{\sigma\kappa}{(1+\sigma\kappa)(1-\rho) + \sigma\kappa(\phi - 1)} v_t^r. \tag{69}$$

for $\sigma\kappa = 1$,

$$\pi_t = \frac{1}{2(1-\rho) + (\phi-1)} v_t^r.$$

$$\sigma^2(\pi_t) = \frac{1}{[2(1-\rho) + (\phi-1)]^2} \frac{1}{1-\rho^2} \sigma_{\varepsilon^r}^2.$$

The backward-looking solution is (8)

$$\pi_{t+1} = \frac{1+\sigma\kappa\phi}{1+\sigma\kappa} \pi_t - \frac{\sigma\kappa}{1+\sigma\kappa} v_t^r + \delta_{t+1}$$

With $\delta_t = 0$, $\phi = 0$,

$$\pi_{t+1} = \frac{1}{1+\sigma\kappa} \pi_t - \frac{\sigma\kappa}{1+\sigma\kappa} v_t^r$$

Working out the variance of an AR(1) with AR(1) shock,

$$(1 - aL)\pi_{t+1} = \frac{k}{1-\rho L} \varepsilon_t^r$$

$$a \equiv \frac{1}{1+\sigma\kappa}; \; k \equiv -\frac{\sigma\kappa}{1+\sigma\kappa}$$

$$\pi_{t+1} = \frac{1}{1-aL} \frac{1}{1-\rho L} k \varepsilon_t^r$$

$$\pi_{t+1} = \frac{1}{a-\rho} \left(\frac{a}{1-aL} - \frac{\rho}{1-\rho L} \right) k \varepsilon_t^r$$

$$\pi_{t+1} = \frac{1}{a-\rho} \left[(a-\rho)\varepsilon_t + (a^2 - \rho^2)\varepsilon_{t-1} + (a^3 - \rho^3)\varepsilon_{t-2} + ... \right] k \varepsilon_t^r$$

$$\sigma^2_{\pi_{t+1}} = \left(\frac{1}{a-\rho}\right)^2 \left[(a-\rho)^2 + (a^2-\rho^2)^2 + (a^3-\rho^3)^2 + ...\right] k^2\sigma^2_{\varepsilon^r_t}$$

$$\sigma^2_{\pi_{t+1}} = \left(\frac{1}{a-\rho}\right)^2 \left[(a^2+\rho^2-2a\rho) + \left(a^4+\rho^4-2a^2\rho^2\right) + (a^6+\rho^6-2a^3\rho^3) + ...)\right] k^2\sigma^2_{\varepsilon^r_t}$$

$$\sigma^2_{\pi_{t+1}} = \left(\frac{1}{a-\rho}\right)^2 \left(\frac{a^2}{1-a^2} + \frac{\rho^2}{1-\rho^2} - 2\frac{a\rho}{1-a\rho}\right) k^2\sigma^2_{\varepsilon^r_t}$$

$$\sigma^2_{\pi_{t+1}} = \frac{(1+a\rho)}{(1-a\rho)}\frac{1}{(1-a^2)(1-\rho^2)}k^2\sigma^2_{\varepsilon^r_t}$$

Note the naive answer is wrong

$$\sigma^2_{\pi_{t+1}} \neq \frac{1}{1-a^2}\sigma^2_{v^r} = \frac{1}{1-a^2}\frac{1}{1-\rho^2}k^2\sigma^2_{\varepsilon^r_t}.$$

Plugging in a and k,

$$\sigma^2_{\pi_{t+1}} = \frac{\left(1+\frac{1}{1+\sigma\kappa}\rho\right)}{\left(1-\frac{1}{1+\sigma\kappa}\rho\right)}\frac{1}{\left(1-\left(\frac{1}{1+\sigma\kappa}\right)^2\right)(1-\rho^2)}\left(\frac{\sigma\kappa}{1+\sigma\kappa}\right)^2\sigma^2_{\varepsilon^v_t}$$

$$\sigma^2_{\pi_{t+1}} = \frac{(1+\sigma\kappa+\rho)}{(1+\sigma\kappa-\rho)}\frac{(\sigma\kappa)^2}{\left((1+\sigma\kappa)^2-1\right)(1-\rho^2)}\sigma^2_{\varepsilon^r_t}$$

For $\sigma\kappa = 1$,

$$\sigma^2_{\pi_{t+1}} = \frac{(2+\rho)}{(2-\rho)}\frac{1}{3(1-\rho^2)}\sigma^2_{\varepsilon^r_t}$$

The ratio of the two solutions is:

$$\frac{\text{backward}}{\text{forward}} = \frac{(2+\rho)}{(2-\rho)}\frac{\left[2(1-\rho)+(\phi-1)\right]^2}{3}.$$

19.2 Fiscal theory formulas for delayed and temporary rate rises

Here I work out the algebra for impulse response functions of the fiscal theory with long term debt model, with an announcement M years ahead of the interest rate rise, and an interest rate rise that only lasts M years, in both continuous and discrete time, Equations (29)-(30).

An interest rate rise from i to i^* that only lasts M years, continuous time:

$$\left[\vartheta \int_0^M e^{-i^* j} e^{-\vartheta j} dj + \vartheta \int_M^\infty e^{-i^* M - i(j-M)} e^{-\vartheta j} dj\right] \frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\vartheta \left[\frac{e^{-(i^*+\vartheta)M}}{i+\vartheta} + \frac{1-e^{-(i^*+\vartheta)M}}{i^*+\vartheta}\right] \frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\frac{P_t^*}{P_t} = \left(\frac{e^{-(i^*+\vartheta)M}}{i+\vartheta} + \frac{1-e^{-(i^*+\vartheta)M}}{i^*+\vartheta}\right) / \left(\frac{1}{i+\vartheta}\right)$$

$$\frac{P_t^*}{P_t} - 1 = \left(1 - e^{-(i^*+\vartheta)M}\right)\left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right) \approx \left(1 - e^{-\vartheta M}\right)\left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right)$$

An announcement of an interest rate rise from i to i^* that starts in M years, continuous time:

$$\left[\vartheta \int_0^M e^{-ij} e^{-\vartheta j} dj + \vartheta \int_M^\infty e^{-iM - i^*(j-M)} e^{-\vartheta j} dj\right] \frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\vartheta \left[\frac{e^{-(i+\vartheta)M}}{i^*+\vartheta} + \frac{1-e^{-(i+\vartheta)M}}{i+\vartheta}\right] \frac{B_t}{P_t^*} = \frac{s}{r}$$

$$\frac{P_t^*}{P_t} = \left(\frac{e^{-(i+\vartheta)M}}{i^*+\vartheta} + \frac{1-e^{-(i+\vartheta)M}}{i+\vartheta}\right) / \left(\frac{1}{i+\vartheta}\right)$$

$$\frac{P_t^*}{P} - 1 = e^{-(i+\vartheta)M}\left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right) \approx e^{-\vartheta M}\left(\frac{i+\vartheta}{i^*+\vartheta} - 1\right)$$

An interest rate rise from i to i^* that only lasts M years, discrete time:

$$\left[\sum_{j=0}^{M-1} \frac{\theta^j}{(1+i^*)^j} + \sum_{j=M}^{\infty} \frac{\theta^M}{(1+i^*)^M} \frac{\theta^{(j-M)}}{(1+i)^{(j-M)}}\right] \frac{B_{t-1}}{P_t^*} = \frac{s}{1-\beta}$$

$$\left[\frac{1 - \left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i^*}} + \frac{\left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i}}\right] \frac{B_{t-1}}{P_t^*} = \frac{s}{1-\beta}$$

Thus,

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{1 - \left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i^*}} + \frac{\left(\frac{\theta}{1+i^*}\right)^M}{1 - \frac{\theta}{1+i}}\right) / \left(\frac{1}{1 - \frac{\theta}{1+i}}\right) - 1$$

$$\frac{P_t^*}{P_t} - 1 = \left(1 - \left(\frac{\theta}{1+i^*}\right)^M\right)\left(\frac{\frac{1}{1-\frac{\theta}{1+i^*}}}{\frac{1}{1-\frac{\theta}{1+i}}} - 1\right)$$

$$\frac{P_t^*}{P_t} - 1 = \left(1 - \left(\frac{\theta}{1+i^*}\right)^M\right)\left(\frac{1+i^*}{1+i_t}\frac{1+i-\theta}{1+i^*-\theta} - 1\right)$$

$$\frac{P_t^*}{P_t} - 1 \approx \left(1 - \theta^M\right)\left(\frac{1+i-\theta}{1+i^*-\theta} - 1\right)$$

An interest rate rise from i to i^* that starts in M years, discrete time:

$$\left[\sum_{j=0}^{M-1}\frac{\theta^j}{(1+i)^j} + \sum_{j=M}^{\infty}\frac{\theta^M}{(1+i)^M}\frac{\theta^{(j-M)}}{(1+i^*)^{(j-M)}}\right]\frac{B_{t-1}}{P_t^*} = \frac{s}{1-\beta}$$

$$\left[\frac{1-\left(\frac{\theta}{1+i}\right)^M}{1-\frac{\theta}{1+i}} + \frac{\left(\frac{\theta}{1+i}\right)^M}{1-\frac{\theta}{1+i^*}}\right]\frac{B_{t-1}}{P_t^*} = \frac{s}{1-\beta}$$

Thus,

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{1-\left(\frac{\theta}{1+i}\right)^M}{1-\frac{\theta}{1+i}} + \frac{\left(\frac{\theta}{1+i}\right)^M}{1-\frac{\theta}{1+i^*}}\right)/\left(\frac{1}{1-\frac{\theta}{1+i}}\right) - 1$$

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{\theta}{1+i}\right)^M\left(\frac{1}{1-\frac{\theta}{1+i^*}} - \frac{1}{1-\frac{\theta}{1+i}}\right)\left(\frac{1}{1-\frac{\theta}{1+i}}\right)$$

$$\frac{P_t^*}{P_t} - 1 = \left(\frac{\theta}{1+i}\right)^M\left(\frac{1+i^*}{1+i}\frac{1+i-\theta}{1+i^*-\theta} - 1\right)$$

$$\frac{P_t^*}{P_t} - 1 \approx \theta^M\left(\frac{1+i-\theta}{1+i^*-\theta} - 1\right)$$

19.3 Sticky-price model solution

Here I derive the explicit solutions (20)-(21), for inflation and output given the equilibrium path of interest rates. The simple model (18)-(19) is

$$x_t = E_t x_{t+1} - \sigma(i_t - E_t\pi_{t+1})$$
$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t.$$

The model with money generalizes the IS equation only, to (33)

$$x_t = E_t x_{t+1} + (\sigma - \xi)\left(\frac{m}{c}\right) E_t\left[\left(i_{t+1} - i^m_{t+1}\right) - \left(i_t - i^m_t\right)\right] - \sigma\left(i_t - E_t\pi_{t+1}\right).$$

We can treat the two cases simultaneously by defining

$$z_t \equiv i_t - \left(\frac{\sigma - \xi}{\sigma}\right)\left(\frac{m}{c}\right) E_t\left[\left(i_{t+1} - i^m_{t+1}\right) - \left(i_t - i^m_t\right)\right]$$

and writing the IS equation as

$$x_t = E_t x_{t+1} - \sigma\left(z_t - E_t\pi_{t+1}\right).$$

One must be careful that lags of z_t are lags of expected interest rate changes, not lags of actual interest rate changes.

Expressing the model in lag operator notation,

$$E_t(1 - L^{-1})x_t = \sigma E_t L^{-1}\pi_t - \sigma z_t$$

$$E_t(1 - \beta L^{-1})\pi_t = \kappa x_t$$

Forward-differencing the second equation,

$$E_t(1 - L^{-1})(1 - \beta L^{-1})\pi_t = E_t(1 - L^{-1})\kappa x_t$$

Then substituting,

$$E_t(1 - L^{-1})\left(1 - \beta L^{-1}\right)\pi_t = \kappa\sigma E_t L^{-1}\pi_t - \kappa\sigma z_t$$

$$E_t\left[(1 - L^{-1})\left(1 - \beta L^{-1}\right) - \kappa\sigma L^{-1}\right]\pi_t = -\kappa\sigma z_t$$

$$E_t\left[1 - (1 + \beta + \kappa\sigma)L^{-1} + \beta L^{-2}\right]\pi_t = -\kappa\sigma z_t.$$

Factor the lag polynomial

$$E_t(1 - \lambda_1 L^{-1})(1 - \lambda_2 L^{-1})\pi_t = -\kappa\sigma z_t$$

where

$$\lambda_i = \frac{(1+\beta+\kappa\sigma) \pm \sqrt{(1+\beta+\kappa\sigma)^2 - 4\beta}}{2}.$$

Since $\lambda_1 > 1$ and $\lambda_2 < 1$, reexpress the result as

$$E_t \left[(1-\lambda_1^{-1}L)(1-\lambda_2 L^{-1})\lambda_1 L^{-1}\pi_t \right] = \kappa\sigma z_t$$

$$E_t \left[(1-\lambda_1^{-1}L)(1-\lambda_2 L^{-1})\pi_{t+1} \right] = \kappa\sigma \lambda_1^{-1} z_t$$

The bounded solutions are

$$\pi_{t+1} = E_{t+1} \frac{\lambda_1^{-1}}{(1-\lambda_1^{-1}L)(1-\lambda_2 L^{-1})} \kappa\sigma z_t + \frac{1}{(1-\lambda_1^{-1}L)} \delta_{t+1}$$

where δ_{t+1} is a sequence of unpredictable random variables, $E_t \delta_{t+1} = 0$. I follow the usual practice and I rule out solutions that explode in the forward direction.

Using a partial fractions decomposition to break up the right hand side,

$$\frac{\lambda_1^{-1}}{\left(1-\lambda_1^{-1}L\right)\left(1-\lambda_2 L^{-1}\right)} = \frac{1}{\lambda_1 - \lambda_2} \left(1 + \frac{\lambda_1^{-1}L}{1-\lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1-\lambda_2 L^{-1}} \right).$$

So,

$$\pi_{t+1} = \frac{1}{\lambda_1 - \lambda_2} E_{t+1} \left(1 + \frac{\lambda_1^{-1}L}{1-\lambda_1^{-1}L} + \frac{\lambda_2 L^{-1}}{1-\lambda_2 L^{-1}} \right) \kappa\sigma z_t + \frac{1}{(1-\lambda_1^{-1}L)} \delta_{t+1}$$

or in sum notation,

$$\pi_{t+1} = \kappa\sigma \frac{1}{\lambda_1 - \lambda_2} \left(z_t + \sum_{j=1}^{\infty} \lambda_1^{-j} z_{t-j} + \sum_{j=1}^{\infty} \lambda_2^{j} E_{t+1} z_{t+j} \right) + \sum_{j=0}^{\infty} \lambda_1^{-j} \delta_{t+1-j}. \tag{70}$$

We can show directly that the long-run impulse-response function is 1:

$$\frac{1}{(1-\lambda_1^{-1})(1-\lambda_2)} \frac{\kappa\sigma}{\lambda_1} = -\frac{\kappa\sigma}{(1-\lambda_1)(1-\lambda_2)}$$

$$= -\frac{\kappa\sigma}{(1-(\lambda_1+\lambda_2)+\lambda_1\lambda_2)} = -\frac{\kappa\sigma}{(1-(1+\beta+\kappa\sigma)+\beta)} = 1.$$

Having found the path of π_t, we can find output by

$$\kappa x_t = \pi_t - \beta E_t \pi_{t+1}.$$

In lag operator notation, and shifting forward one period,

$$\kappa x_{t+1} = E_{t+1}\left[(1-\beta L^{-1})\pi_{t+1}\right]$$

$$\kappa x_{t+1} = \frac{\kappa\sigma}{\lambda_1-\lambda_2}E_{t+1}\left[(1-\beta L^{-1})\left(1+\frac{\lambda_1^{-1}L}{1-\lambda_1^{-1}L}+\frac{\lambda_2 L^{-1}}{1-\lambda_2 L^{-1}}\right)z_t\right]+E_{t+1}\frac{(1-\beta L^{-1})}{(1-\lambda_1^{-1}L)}\delta_{t+1}.$$

We can rewrite the polynomials to give

$$\kappa x_{t+1} = \frac{\kappa\sigma}{\lambda_1-\lambda_2}E_{t+1}\left[\frac{1-\beta\lambda_1^{-1}}{1-\lambda_1^{-1}L}+\frac{\left(1-\beta\lambda_2^{-1}\right)\left(\lambda_2 L^{-1}\right)}{1-\lambda_2 L^{-1}}\right]z_t + E_{t+1}\left[\frac{1-\beta\lambda_1^{-1}}{1-\lambda_1^{-1}L}\right]\delta_{t+1}.$$

(In the second term, I use $E_t\left[\beta L^{-1}\delta_{t+1}\right]=0$) or, in sum notation,

$$\kappa x_{t+1} = \frac{\kappa\sigma}{\lambda_1-\lambda_2}\left[\left(1-\beta\lambda_1^{-1}\right)\sum_{j=0}^{\infty}\lambda_1^{-j}z_{t-j}+\left(1-\beta\lambda_2^{-1}\right)\sum_{j=1}^{\infty}\lambda_2^{j}E_{t+1}z_{t+j}\right]+$$

$$+\left(1-\beta\lambda_1^{-1}\right)\sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}.$$

19.4 Model and solutions in continuous time

It is convenient to work both in discrete and continuous time. To keep the math simple, I consider the perfect-foresight continuous-time specification, and treat the impulse response function as a once and for all unexpected shock. The continuous time version of the model is

$$\frac{dx_t}{dt} = \sigma\left(i_t - \pi_t\right) \tag{71}$$
$$\frac{d\pi_t}{dt} = \rho\pi_t - \kappa x_t \tag{72}$$

The solution is

$$\pi_t = \pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\sqrt{\rho^2+4\kappa\sigma}}\left[\int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right] \tag{73}$$

$$\kappa x_t = \lambda_1 \pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\sqrt{\rho^2+4\kappa\sigma}}\left[\lambda_1 \int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds - \lambda_2 \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right] \tag{74}$$

where

$$\lambda_1 = \frac{1}{2}\left(\sqrt{\rho^2+4\kappa\sigma}+\rho\right)$$
$$\lambda_2 = \frac{1}{2}\left(\sqrt{\rho^2+4\kappa\sigma}-\rho\right)$$

Here I have defined the roots with the square root term first, so both are positive numbers. That choice clarifies the notation for continuous time expressions. However, it means that $\lambda_1+\lambda_2$ shows up where corresponding discrete time expressions have $\lambda_1-\lambda_2$, which can cause some confusion.

To derive the solution we proceed as in discrete time. Difference (71),

$$\frac{d^2\pi_t}{dt^2} = \rho\frac{d\pi_t}{dt} + \kappa\sigma\pi_t - \kappa\sigma i_t$$

$$\left(\frac{d^2\pi_t}{dt^2} - \rho\frac{d\pi_t}{dt} - \kappa\sigma\pi_t\right) = -\kappa\sigma i_t.$$

We seek roots of the form

$$\left(\frac{d}{dt}-\lambda_1\right)\left(\frac{d}{dt}+\lambda_2\right)\pi_t = -\kappa\sigma i_t \tag{75}$$

in which case

$$\frac{d^2\pi_t}{dt^2} + (\lambda_2-\lambda_1)\frac{d\pi_t}{dt} - \lambda_1\lambda_2\pi_t = -\kappa\sigma i_t. \tag{76}$$

Matching coefficients,

$$\lambda_1\lambda_2 = \kappa\sigma$$
$$\lambda_1-\lambda_2 = \rho.$$

Solving,

$$\lambda_1 - \frac{\kappa\sigma}{\lambda_1} = \rho$$
$$\lambda_1^2 - \rho\lambda_1 - \kappa\sigma = 0,$$

and hence,

$$\lambda_1 = \frac{1}{2}\left(\sqrt{\rho^2 + 4\kappa\sigma} + \rho\right)$$
$$\lambda_2 = \lambda_1 - \rho = \frac{1}{2}\left(\sqrt{\rho^2 + 4\kappa\sigma} - \rho\right).$$

The solution to (76) is derived in Cochrane (2014c),

$$\pi_t = \pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\lambda_2 + \lambda_1}\left[\int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$

It straightforward to check. The derivatives are

$$\frac{d\pi_t}{dt} = -\lambda_2 \pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\lambda_2 + \lambda_1}\left[-\lambda_2 \int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \lambda_1 \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$

$$\frac{d^2\pi_t}{dt^2} = \lambda_2^2 \pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\lambda_2 + \lambda_1}\left[-(\lambda_1 + \lambda_2) i_t + \lambda_2^2 \int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \lambda_1^2 \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$

Plugging these derivatives in to the differential equation (75), we have

$$\lambda_2^2 \pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\lambda_2 + \lambda_1}\left[-(\lambda_1 + \lambda_2) i_t + \lambda_2^2 \int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \lambda_1^2 \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$
$$+ \rho\lambda_2 \pi_0 e^{-\lambda_2 t} - \frac{\rho\kappa\sigma}{\lambda_2 + \lambda_1}\left[-\lambda_2 \int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \lambda_1 \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$
$$- \kappa\sigma\pi_0 e^{-\lambda_2 t} - \frac{(\kappa\sigma)^2}{\lambda_2 + \lambda_1}\left[\int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right] + \kappa\sigma i_t =?0.$$

Collecting terms, the equation holds if

$$\lambda_2^2 + \rho\lambda_2 - \kappa\sigma = 0$$
$$\lambda_1^2 - \rho\lambda_1 - \kappa\sigma = 0$$

which follow by substitution.

We can find the output response from

$$\kappa x_t = \rho\pi_t - \frac{d\pi_t}{dt} =$$
$$\rho\pi_0 e^{-\lambda_2 t} + \frac{\rho\kappa\sigma}{\lambda_2+\lambda_1}\left[\int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$
$$+\lambda_2\pi_0 e^{-\lambda_2 t} - \frac{\kappa\sigma}{\lambda_1+\lambda_2}\left[-\lambda_2\int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + \lambda_1\int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$

Collecting terms

$$\kappa x_t = \lambda_1\pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\lambda_2+\lambda_1}\left[(\rho+\lambda_2)\int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds + (\rho-\lambda_1)\int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right]$$

or,

$$\kappa x_t = \lambda_1\pi_0 e^{-\lambda_2 t} + \frac{\kappa\sigma}{\sqrt{\rho^2+4\kappa\sigma}}\left[\lambda_1\int_{s=0}^{t} e^{-\lambda_2(t-s)} i_s ds - \lambda_2\int_{s=t}^{\infty} e^{-\lambda_1(s-t)} i_s ds\right].$$

19.5 Impulse response function – explicit solution

The solution (70) is

$$\pi_{t+1} = \frac{\kappa\sigma}{\lambda_1-\lambda_2}\left(i_t + \sum_{j=1}^{\infty}\lambda_1^{-j} i_{t-j} + E_{t+1}\sum_{j=1}^{\infty}\lambda_2^{j} i_{t+j}\right) + \sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}$$

$$\lambda_1 = \frac{(1+\beta+\kappa\sigma)+\sqrt{(1+\beta+\kappa\sigma)^2-4\beta}}{2}$$
$$\lambda_1 = \frac{(1+\beta+\kappa\sigma)-\sqrt{(1+\beta+\kappa\sigma)^2-4\beta}}{2}$$

While it is straightforward to calculate and simulate the solution for a given path of interest rates, it is useful also to have a formula for the response to a step function. We want to find the impulse-response function to $i_t = 0$, $t < 0$, and $i_t = i$, $t = 0, 1, 2, ...$ The interest rate rise is announced at time $-M$, so only $\delta_{-M} \neq 0$. That response is,

$$t < -(M+1): \; \pi_{t+1} = 0$$

$$-(M+1) \leq t \leq 0: \pi_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left(\frac{\lambda_2^{-t}}{1-\lambda_2}\right) + \lambda_1^{-(t+1+M)}\delta_{-M}$$

$$0 < t: \pi_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left(\frac{1}{1-\lambda_2} + \frac{\lambda_1^{-1}(1-\lambda_1^{-t})}{1-\lambda_1^{-1}}\right) + \lambda_1^{-(t+1+M)}\delta_{-M}$$

Proceeding in the same way, the solution for x is

$$\kappa x_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left((1-\beta\lambda_1^{-1})\sum_{j=0}^{\infty}\lambda_1^{-j} i_{t-j} + (1-\beta\lambda_2^{-1})E_{t+1}\sum_{j=1}^{\infty}\lambda_2^{j} i_{t+j}\right) + (1-\beta\lambda_1^{-1})\sum_{j=0}^{\infty}\lambda_1^{-j}\delta_{t+1-j}$$

so the impulse-response function to $i_t = 0$, $t < 0$, and $i_t = i$, $t = 0, 1, 2, ..$announced at time $-M$, is,

$$t < -(M+1): \; x_{t+1} = 0$$

$$-(M+1) \leq t \leq -1: \kappa x_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}(1-\beta\lambda_2^{-1})\frac{\lambda_2^{-t+1}}{1-\lambda_2} + (1-\beta\lambda_1^{-1})\lambda_1^{-(t+1+M)}\delta_{-M}$$

$$0 \leq t: \kappa x_{t+1} = \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left((1-\beta\lambda_1^{-1})\frac{1-\lambda_1^{-(t+1)}}{1-\lambda_1^{-1}} + (1-\beta\lambda_2^{-1})\frac{\lambda_2}{1-\lambda_2}\right) + \lambda_1^{-(t+1+M)}\delta_{-M}$$

The interest rate is then

$$r_t = i_t - E_t\pi_{t+1}.$$

For the impulse-response function, the expected and actual values are the same, except at $-M$, where though $\pi_{-M} \neq 0$, $E_{-M-1}\pi_{-M} = 0$. Hence,

$$t \leq -(M+1): \; r_t = 0 \tag{77}$$

$$-M \leq t < 0 : r_t = -\frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left(\frac{\lambda_2^{-t}}{1-\lambda_2}\right) - \lambda_1^{-(t+1+M)}\delta_{-M} \tag{78}$$

$$t = 0 : r_t = i - \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left(\frac{\lambda_2^{-t}}{1-\lambda_2}\right) - \lambda_1^{-(t+1+M)}\delta_{-M} \tag{79}$$

$$0 < t : r_t = i - \frac{\kappa\sigma}{\lambda_1 - \lambda_2}\left(\frac{1}{1-\lambda_2} + \frac{\lambda_1^{-1}(1-\lambda_1^{-t})}{1-\lambda_1^{-1}}\right) - \lambda_1^{-(t+1+M)}\delta_{-M} \tag{80}$$

19.6 Impulse-response with long-term debt and price stickiness

I develop the exact nonlinear formulas for the value of surpluses and a linear approximation. The linear approximation turns out to be quite accurate in this application.

An interest rate rise from time $t = 0$ onwards is announced at time $t = -M$. I calculate for each value of the inflation shock δ_{-M} the percent change in a constant surplus corresponding to that shock. Writing the surplus as $se^{\Delta s}$, the value of nominal debt before the shock satisfies

$$\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{1+f^{(k)}}\right)\frac{B^{(j)}}{P_{-M}} = \sum_{t=-M}^{\infty}\beta^{t-M}s. = \frac{1}{1-\beta}s.$$

where $\{B^{(j)}\}$ is the observed maturity structure of the debt, and the observed forward rates are $f^{(j)}$, ($f_t^{(j)}$ is the forward rate at time t for loans from $t+j$ to $t+j+1$; $f_t^{(0)} = i_t$ is the one-period interest rate). After the shock, nominal interest rates increase by i, the price level jumps from P_{-M} to P^*_{-M}, with

$$e^{\pi^*_{-M}} = e^{\pi_{-M}+\delta_{-M}} = \frac{P^*_{-M}}{P_{-M}}.$$

Here, π_{-M} denotes the solution with $\delta = 0$, so actual inflation after the shock is announced is $\pi^*_{-M} = \pi_{-M} + \delta_{-M}$. The basic solution for inflation (70) includes a jump in inflation when the shock is announced, and I have defined δ as additional unexpected changes in inflation. Surpluses rise to $se^{\Delta s}$, giving

$$\sum_{j=0}^{\infty}\left(\prod_{k=0}^{M-1}\frac{1}{1+f^{(k)}}\right)\left(\prod_{k=M}^{j-1}\frac{1}{1+f^{(k)}+i}\right)\frac{B^{(j)}}{P^{*}_{-M}}=\sum_{t=-M}^{\infty}\beta^{(t-M)}\frac{u'(C_t)}{u'(C_{-M})}se^{\Delta s} \tag{81}$$

To easily calculate the multiperiod discount factor on the right hand side, I use

$$\frac{u'(C_t)}{u'(C_\tau)}=\frac{e^{-\gamma(c+x_t)}}{e^{-\gamma(c+x_\tau)}}=e^{-\frac{1}{\sigma}(x_t-x_\tau)}$$

Dividing pre and post shock values of (81), s cancels and

$$e^{\pi_{-M}+\delta_{-M}}=\frac{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{M-1}\frac{1}{1+f^{(k)}}\right)\left(\prod_{k=M}^{j-1}\frac{1}{1+f^{(k)}+i}\right)B^{(j)}}{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{1+f^{(k)}}\right)B^{(j)}}\frac{\sum_{t=-M}^{\infty}\beta^{(t-M)}}{\sum_{t=-M}^{\infty}\beta^{(t-M)}e^{-\frac{1}{\sigma}(x_t-x_{-M})}}e^{-\Delta s}.$$

Conversely, then, we can find the surplus required to support a given time -M shock δ_{-M} – whether that surplus comes from active or from passive fiscal policy – by solving for Δs,

$$e^{\Delta s}=\frac{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{M-1}\frac{1}{1+f^{(k)}}\right)\left(\prod_{k=M}^{j-1}\frac{1}{1+f^{(k)}+i}\right)B^{(j)}}{\sum_{j=0}^{\infty}\left(\prod_{k=0}^{j-1}\frac{1}{1+f^{(k)}}\right)B^{(j)}}\frac{\sum_{t=-M}^{\infty}\beta^{(t-M)}}{\sum_{t=-M}^{\infty}\beta^{(t-M)}e^{-\frac{1}{\sigma}(x_t-x_{-M})}}e^{-(\pi_{-M}+\delta_{-M})}. \tag{82}$$

For each choice of δ_{-M}, then, I find the solution for inflation and interest rates by (78)-(80); I compute the product of real rates in the bottom right term of (82), and I compute the required percentage change in surplus Δs. To find the fiscal-theory / long-term debt solution, I search for the δ_{-M} that produces $\Delta s = 0$. It is important to treat the numerator and denominator of the last term of (82) equally. If one truncates the denominator, truncate the numerator at the same point.

19.7 Linearized valuation equation

To linearly approximate (82), write

$$e^{\Delta s} \approx V \frac{\sum_{t=-M}^{\infty} \beta^{(t-M)}}{\sum_{t=-M}^{\infty} \beta^{(t-M)} e^{-\frac{1}{\sigma}(x_t - x_{-M})}} e^{-(\pi_{-M}+\delta_{-M})}. \tag{83}$$

$$1+\Delta s \approx (1+v) \frac{\sum_{t=-M}^{\infty} \beta^{(t-M)}}{\sum_{t=-M}^{\infty} \beta^{(t-M)} \left(1-\frac{1}{\sigma}(x_t - x_{-M})\right)} (1-(\pi_{-M}+\delta_{-M})). \tag{84}$$

$$1+\Delta s \approx (1+v) \frac{1}{1-\frac{\sum_{t=-M}^{\infty} \beta^{(t-M)} \frac{1}{\sigma}(x_t - x_{-M})}{\sum_{t=-M}^{\infty} \beta^{(t-M)}}} (1-(\pi_{-M}+\delta_{-M})). \tag{85}$$

$$\Delta s \approx v + \frac{\sum_{t=-M}^{\infty} \beta^{(t-M)} \frac{1}{\sigma}(x_t - x_{-M})}{\sum_{t=-M}^{\infty} \beta^{(t-M)}} - (\pi_{-M}+\delta_{-M}). \tag{86}$$

$$\Delta s \approx v + (1-\beta) \sum_{t=-M}^{\infty} \beta^{(t-M)} \frac{1}{\sigma}(x_t - x_{-M}) - (\pi_{-M}+\delta_{-M}). \tag{87}$$

In numerical experimentation, it turns out that the exact and linearized approach produce almost exactly the same answer to the first few decimals. So, the nonlinearity of long-term present values is not an issue for this magnitude – a few percent at most – of interest rate variation.

19.8 The Model with Money

This section derives the model with money (33). The utility function is

$$\max E \int_{t=0}^{\infty} e^{-\delta t} u(c_t, M_t/P_t) dt.$$

The present-value budget constraint is

$$\frac{B_0 + M_0}{P_0} = \int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s ds} \left[c_t - y_t + s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right] dt$$

where

$$r_t = i_t - \frac{dP_t}{P_t}$$

and s denotes real net taxes paid, and thus the real government primary surplus. This budget constraint is the present value form of

$$d(B_t + M_t) = i_t B_t + i_t^m M_t + P_t(y_t - c_t - s_t).$$

Introducing a multiplier λ on the present value budget constraint, we have

$$\frac{\partial}{\partial c_t} : e^{-\delta t} u_c(t) = \lambda e^{-\int_{s=0}^{t} r_s ds}.$$

where (t) means $(c_t, M_t/P_t)$. Differentiating with respect to time,

$$-\delta e^{-\delta t} u_c(t) + e^{-\delta t} u_{cc}(t) \frac{dc_t}{dt} + e^{-\delta t} u_{cm}(t) \frac{dm_t}{dt} = -\lambda r_t \, e^{-\int_{s=0}^{t} r_s ds}$$

where $m_t \equiv M_t/P_t$. Dividing by $e^{-\delta t} u_c(t)$, we obtain the intertemporal first order condition:

$$-\frac{c_t u_{cc}(t)}{u_c(t)} \frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{dm_t}{m_t} = (r_t - \delta) \, dt. \tag{88}$$

The first-order condition with respect to M is

$$\begin{aligned}
\frac{\partial}{\partial M_t} : e^{-\delta t} u_m(t) \frac{1}{P_t} &= \lambda e^{-\int_{s=0}^{t} r_s ds} (i_t - i_t^m) \frac{1}{P_t} \\
e^{-\delta t} u_m(t) &= e^{-\delta t} u_c(t) (i_t - i_t^m) \\
\frac{u_m(t)}{u_c(t)} &= i_t - i_t^m. \qquad (89)
\end{aligned}$$

The last equation is the usual money demand curve.

Thus, an equilibrium $c_t = y_t$ satisfies

$$\frac{-c_t u_{cc}(t)}{u_c(t)} \frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)} \frac{dm_t}{m_t} = -\delta dt + \left(i_t - \frac{dP_t}{P_t} \right) dt \tag{90}$$

$$\frac{u_m(t)}{u_c(t)} = i_t - i_t^m \tag{91}$$

$$\frac{B_0 + M_0}{P_0} = \int_{t=0}^{\infty} e^{-\int_{s=0}^{t} r_s ds} \left[s_t + (i_t - i_t^m) \frac{M_t}{P_t} \right] dt \tag{92}$$

The last equation combines the consumer's budget constraint and equilibrium $c = y$. I call it the government debt valuation formula.

19.8.1 CES functional form

I use a standard money in the utility function specification with a CES functional form,

$$u(c_t, m_t) = \frac{1}{1-\gamma}\left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]^{\frac{1-\gamma}{1-\theta}}.$$

I use the notation $m = M/P$, with capital letters for nominal and lowercase letters for real quantities.

This CES functional form nests three important special cases. Perfect substitutes is the case $\theta = 0$:

$$u(c_t, m_t) = \frac{1}{1-\gamma}\left[c_t + \alpha m_t\right]^{1-\gamma}.$$

The Cobb-Douglas case is $\theta \longrightarrow 1$:

$$u(c_t, m_t) \longrightarrow \frac{1}{1-\gamma}\left[c_t^{\frac{1}{1+\alpha}} m_t^{\frac{\alpha}{1+\alpha}}\right]^{1-\gamma}. \tag{93}$$

The monetarist limit is $\theta \to \infty$:

$$u(c_t, m_t) \to \frac{1}{1-\gamma}\left[\min\left(c_t, \alpha m_t\right)\right]^{1-\gamma}.$$

I call it the monetarist limit because money demand is then $M_t/P_t = c_t/\alpha$, i.e. $\alpha = 1/V$ is constant, and the interest elasticity is zero. The separable case is $\theta = \gamma$:

$$u(c_t, m_t) = \frac{1}{1-\gamma}\left[c_t^{1-\gamma} + \alpha m_t^{1-\gamma}\right].$$

In the separable case, u_c is independent of m, so money has no effect on the intertemporal substitution relation, and hence on inflation and output dynamics in a new-Keynesian model under an interest rate target. Terms in $(\theta - \gamma)$ or $(\sigma - \xi)$ with $\sigma = 1/\gamma$ and $\xi = 1/\theta$ will characterize deviations from the separable case, how much the marginal utility of consumption is affected by money.

With this functional form, the derivatives are

$$u_c = \left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]^{\frac{\theta-\gamma}{1-\theta}} c_t^{-\theta}$$

$$u_m = \left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]^{\frac{\theta-\gamma}{1-\theta}} \alpha m_t^{-\theta}.$$

Equilibrium condition (91) becomes

$$\frac{u_m(t)}{u_c(t)} = \alpha \left(\frac{m_t}{c_t}\right)^{-\theta} = i_t - i_t^m. \tag{94}$$

The second derivative with respect to consumption is

$$\frac{u_{cc}}{u_c} = (\theta-\gamma)\frac{1}{\left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]} c_t^{-\theta} - \theta c_t^{-1}$$

$$-\frac{cu_{cc}}{u_c} = -\frac{(\theta-\gamma)c_t^{1-\theta} - \theta\left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]}{\left[c_t^{1-\theta} + \alpha m_t^{1-\theta}\right]}$$

$$-\frac{cu_{cc}}{u_c} = \frac{\gamma c_t^{1-\theta} + \theta\alpha m_t^{1-\theta}}{c_t^{1-\theta} + \alpha m_t^{1-\theta}}$$

$$-\frac{cu_{cc}}{u_c} = \gamma\frac{\left[1 + \frac{\theta}{\gamma}\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}\right]}{\left[1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}\right]}.$$

The cross derivative is

$$\begin{aligned}\frac{mu_{cm}}{u_c} &= (\theta-\gamma)\frac{\alpha m_t^{1-\theta}}{c_t^{1-\theta} + \alpha m_t^{1-\theta}} \\ &= (\theta-\gamma)\frac{\alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha\left(\frac{m_t}{c_t}\right)^{1-\theta}}.\end{aligned}$$

or, using (94)

$$\frac{mu_{cm}}{u_c} = (\theta-\gamma)\frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}.$$

19.8.2 Money demand

Money demand (94) can be written

$$\frac{m_t}{c_t} = \left(\frac{1}{\alpha}\right)^{-\xi} (i_t - i_t^m)^{-\xi}. \tag{95}$$

where $\xi = 1/\theta$ becomes the interest elasticity of money demand, in log form, and α governs the overall level of money demand.

The steady state obeys

$$\frac{m}{c} = \left(\frac{1}{\alpha}\right)^{-\xi} (i - i^m)^{-\xi}. \tag{96}$$

so we can write money demand (95) in terms of steady state real money as

$$\frac{m_t}{c_t} = \left(\frac{m}{c}\right)\left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}, \tag{97}$$

avoiding the parameter α. (Throughout, numbers without time subscripts denote steady state values.)

The product $\frac{m}{c}(i - i^m)$, the interest cost of holding money, appears in many subsequent expressions. It is

$$\frac{m}{c}(i - i^m) = \left(\frac{1}{\alpha}\right)^{-\xi} (i - i^m)^{1-\xi}.$$

With $\xi < 1$, as interest rates go to zero this interest cost goes to zero as well.

19.8.3 Intertemporal Substitution

The first order condition for the intertemporal allocation of consumption (90) is

$$-\frac{c_t u_{cc}(t)}{u_c(t)}\frac{dc_t}{c_t} - \frac{m_t u_{cm}(t)}{u_c(t)}\frac{dm_t}{m_t} = -\delta dt + (i_t - \pi_t)\, dt$$

where $\pi_t = dP_t/P_t$ is inflation. This equation shows us how, with nonseparable utility, monetary policy can distort the allocation of consumption over time, in a way not captured by the usual interest rate effect. That is the central goal here. In the case of complements, $u_{cm} > 0$ (more money raises the marginal utility of consumption), larger money growth makes it easier to consume in the future relative to the present, and acts like a higher interest rate, inducing higher consumption growth.

Substituting in the CES derivatives,

$$\gamma \frac{1 + \frac{\theta}{\gamma}\alpha \left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha \left(\frac{m_t}{c_t}\right)^{1-\theta}} \frac{dc_t}{c_t} - (\theta - \gamma) \frac{\alpha \left(\frac{m_t}{c_t}\right)^{1-\theta}}{1 + \alpha \left(\frac{m_t}{c_t}\right)^{1-\theta}} \frac{dm_t}{m_t} = -\delta dt + (i_t - \pi_t)\, dt$$

and using (94) to eliminate α

$$\gamma \frac{1 + \frac{\theta}{\gamma} \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \frac{dc_t}{c_t} - (\theta - \gamma) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \frac{dm_t}{m_t} = -\delta dt + (i_t - \pi_t)\, dt \tag{98}$$

We can make this expression prettier as

$$\gamma \frac{dc_t}{c_t} + (\theta - \gamma) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = -\delta dt + (i_t - \pi_t)\, dt$$

Rexpressing in terms of the intertemporal substitution elasticity $\sigma = 1/\gamma$ and interest elasticity of money demand $\xi = 1/\theta$, and multiplying by σ,

$$\frac{dc_t}{c_t} + \left(\frac{\sigma - \xi}{\xi}\right) \frac{\left(\frac{m_t}{c_t}\right)(i_t - i_t^m)}{1 + \left(\frac{m_t}{c_t}\right)(i_t - i_t^m)} \left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = -\delta \sigma dt + \sigma (i_t - \pi_t)\, dt. \tag{99}$$

We want to substitute interest rates for money. To that end, differentiate the money demand curve

$$\frac{m_t}{c_t} = \left(\frac{m}{c}\right) \left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi}$$

$$\frac{m_t}{c_t} \left(\frac{dm_t}{m_t} - \frac{dc_t}{c_t}\right) = -\xi \left(\frac{m}{c}\right) \left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m}$$

$$\left(\frac{dc_t}{c_t} - \frac{dm_t}{m_t}\right) = \xi \frac{\frac{m}{c}}{\frac{m_t}{c_t}} \left(\frac{i_t - i_t^m}{i - i^m}\right)^{-\xi} \frac{d(i_t - i_t^m)}{i_t - i_t^m}$$

Substituting,

$$\frac{dc_t}{c_t}+\left(\frac{\sigma-\xi}{\xi}\right)\frac{\left(\frac{m_t}{c_t}\right)\left(i_t-i_t^m\right)}{1+\left(\frac{m_t}{c_t}\right)\left(i_t-i_t^m\right)}\left(\xi\frac{\frac{m}{c}}{\frac{m_t}{c_t}}\left(\frac{i_t-i_t^m}{i-i^m}\right)^{-\xi}\frac{d\left(i_t-i_t^m\right)}{i_t-i_t^m}\right)=-\delta\sigma dt+\sigma\left(i_t-\pi_t\right)dt.$$

$$\frac{dc_t}{c_t}+(\sigma-\xi)\frac{m}{c}\frac{1}{1+\left(\frac{m_t}{c_t}\right)\left(i_t-i_t^m\right)}\left(\frac{i_t-i_t^m}{i-i^m}\right)^{-\xi}d\left(i_t-i_t^m\right)=-\delta\sigma dt+\sigma\left(i_t-\pi_t\right)dt.$$

With $x_t=\log c_t$, $dx_t=dc_t/c_t m$, approximating around a steady state, and approximating that the interest cost of holding money is small, $\left(\frac{m}{c}\right)\left(i-i^m\right) << 1$, we obtain the intertemporal substitution condition modified by interest costs,

$$\frac{dx_t}{dt}+(\sigma-\xi)\frac{m}{c}\frac{d\left(i_t-i_t^m\right)}{dt}=\sigma\left(i_t-\pi_t\right). \tag{100}$$

In discrete time,

$$E_t x_{t+1}-x_t+(\sigma-\xi)\left(\frac{m}{c}\right)\left[E_t\left(i_{t+1}-i_{t+1}^m\right)-\left(i_t-i_t^m\right)\right]=\sigma\left(i_t-E_t\pi_{t+1}\right).$$

For models with monetary control, one wants an IS curve expressed in terms of the monetary aggregate. From (99), with the same approximations and $\tilde{m}=\log(m)$,

$$\frac{dx_t}{dt}+\left(\frac{\sigma-\xi}{\xi}\right)\left(\frac{m}{c}\right)\left(i-i^m\right)\left(\frac{dx_t}{dt}-\frac{d\tilde{m}_t}{dt}\right)=\sigma\left(i_t-\pi_t\right)dt. \tag{101}$$

In discrete time,

$$\left(E_t x_{t+1}-x_t\right)+\left(\frac{\sigma-\xi}{\xi}\right)\left(\frac{m}{c}\right)\left(i-i^m\right)\left[\left(E_t x_{t+1}-x_t\right)-E_t\left(\tilde{m}_{t+1}-\tilde{m}_t\right)\right]=\sigma\left(i_t-\pi_t\right). \tag{102}$$

19.9 Three-equation model solution

I solve the three-equation model of Figure 25 by standard methods, incorporating the Taylor rule in to monetary policy rather than conditioning on the equilibrium interest rate and then constructing the underlying Taylor rule. Both methods give the same answer, but a conventional calculation is more transparent in this case, and it verifies that both approaches give the same answer. This is a much condensed version of the treatment in Cochrane (2016a).

While one can solve the model quickly via matrix techniques, here I use lag operator techniques to write the solution for inflation analytically.

The model is

$$
\begin{aligned}
x_t &= E_t x_{t+1} - \sigma(i_t - E_t \pi_{t+1}) \\
\pi_t &= \beta E_t \pi_{t+1} + \kappa x_t \\
i_t &= \phi \pi_t + \hat{v}_t^i \\
v_t^i &= \rho v_{t-1}^i + \epsilon_t^i
\end{aligned}
$$

Substituting the Taylor rule,

$$
\begin{aligned}
x_t &= E_t x_{t+1} - \sigma(\phi \pi_t + v_t^i - E_t \pi_{t+1}) \\
\pi_t &= \beta^f E_t \pi_{t+1} + \kappa x_t
\end{aligned}
$$

Expressing the model in lag operator notation,

$$
E_t(1 - L^{-1}) x_t = \sigma E_t \left(L^{-1} - \phi \right) \pi_t - \sigma v_t^i
$$

$$
E_t(1 - \beta^f L^{-1}) \pi_t = \kappa x_t
$$

Forward-differencing the second equation,

$$
E_t(1 - M L^{-1})(1 - \beta^f L^{-1}) \pi_t = E_t(1 - M L^{-1}) \kappa x_t
$$

Then substituting into the first equation,

$$
E_t(1 - M L^{-1}) \left(1 - \beta^f L^{-1} \right) \pi_t = \kappa \sigma E_t \left(L^{-1} - \phi \right) \pi_t - \kappa \sigma v_t^i
$$

$$
E_t \left[1 - \frac{M + \beta^f + \kappa\sigma}{1 + \kappa\sigma\phi} L^{-1} + \frac{M \beta^f}{1 + \kappa\sigma\phi} L^{-2} \right] \pi_t = -\frac{\kappa\sigma}{1 + \kappa\sigma\phi} v_t^i.
$$

Factor the lag polynomial

$$E_t(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})\pi_t = -\frac{\kappa\sigma}{1+\kappa\sigma\phi}v_t^i$$

where

$$\lambda = \frac{M+\beta^f+\kappa\sigma \pm \sqrt{\left(M+\beta^f+\kappa\sigma\right)^2 - 4M\beta^f\left(1+\phi\kappa\sigma\right)}}{2\left(1+\kappa\sigma\phi\right)}$$

These lag operator roots are the inverse of the eigenvalues of the usual transition matrix. The system is stable and solved backward for $\lambda > 1$; it is unstable and solved forward for $\lambda < 1$.

The standard three-equation model uses $\phi > 1$ so both roots are unstable, $\lambda_1 < 1$ and $\lambda_2 < 1$. Then, we can write

$$E_t(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})\pi_t = -\frac{\kappa\sigma}{1+\kappa\sigma\phi}v_t^i$$

$$\pi_t = -E_t\frac{1}{(1-\lambda_1 L^{-1})(1-\lambda_2 L^{-1})}\frac{\kappa\sigma}{1+\kappa\sigma\phi}v_t^i$$

$$\pi_t = E_t\frac{1}{\lambda_1-\lambda_2}\left(\frac{-\lambda_1}{1-\lambda_1 L^{-1}}+\frac{\lambda_2}{1-\lambda_2 L^{-1}}\right)\frac{\kappa\sigma}{1+\kappa\sigma\phi}v_t^i$$

$$\pi_t = \frac{\kappa\sigma}{1+\kappa\sigma\phi}\frac{1}{\lambda_1-\lambda_2}E_t\left(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j v_{t+j}^i+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j v_{t+j}^i\right)$$

Using the AR(1) form of the disturbance v^i,

$$\pi_t = \frac{\kappa\sigma}{1+\kappa\sigma\phi}\frac{1}{\lambda_1-\lambda_2}\left(-\lambda_1\sum_{j=0}^{\infty}\lambda_1^j\rho^j+\lambda_2\sum_{j=0}^{\infty}\lambda_2^j\rho^j\right)\hat{\imath}_t$$

$$\pi_t = \frac{\kappa\sigma}{1+\kappa\sigma\phi}\frac{1}{\lambda_1-\lambda_2}\left(-\frac{\lambda_1}{1-\lambda_1\rho}+\frac{\lambda_2}{1-\lambda_2\rho}\right)v_t^i$$

$$\pi_t = \frac{\kappa\sigma}{1+\kappa\sigma\phi}\frac{1}{\lambda_1-\lambda_2}\left(\frac{\lambda_2\left(1-\lambda_1\rho\right)-\lambda_1\left(1-\lambda_2\rho\right)}{\left(1-\lambda_1\rho\right)\left(1-\lambda_2\rho\right)}\right)v_t^i$$

$$\pi_t = -\frac{\kappa\sigma}{1+\kappa\sigma\phi}\left(\frac{1}{\left(1-\lambda_1\rho\right)\left(1-\lambda_2\rho\right)}\right)v_t^i$$

Thus, to produce Figure 25, I simply simulate the AR(1) impulse-response, for $\{v_t^i\}$, calculate π_t by the last equation, and calculate $i_t = \phi\pi_t + v_t^i$.