The constraint on public debt when $r < g$ but $g < m$

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Abstract

In most advanced economies over the past decade real interest rates have been decidedly below the growth rate of the economy, while estimates of the marginal product of capital remain above it. Thus, there is a finite bubble component in public debt, and the economy is dynamically efficient. This paper derives nine lessons for how this affects the constraints on public spending and public debt: (i) one-time deficit gambles are feasible, (ii) the present value of primary surpluses can be lower than the outstanding debt, (iii) the government can run a perpetual deficit, (iv) the maximum size of the public deficit as a ratio of private assets is equal to the growth rate minus the interest rate on the public debt, (v) the government can increase public spending and pay for it by collecting a bubble premium on the debt, but this lowers the interest rate, increases inequality, and has an upper bound that is lower in more financially developed economies, (vi) redistributive policies shrink the feasible amount of persistent public spending, (vii) income tax cuts pay for themselves, up to a limit, (viii) inflation volatility reduces the fiscal space available for spending, and (ix) financial repression allows the government to spend more.

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1 Introduction

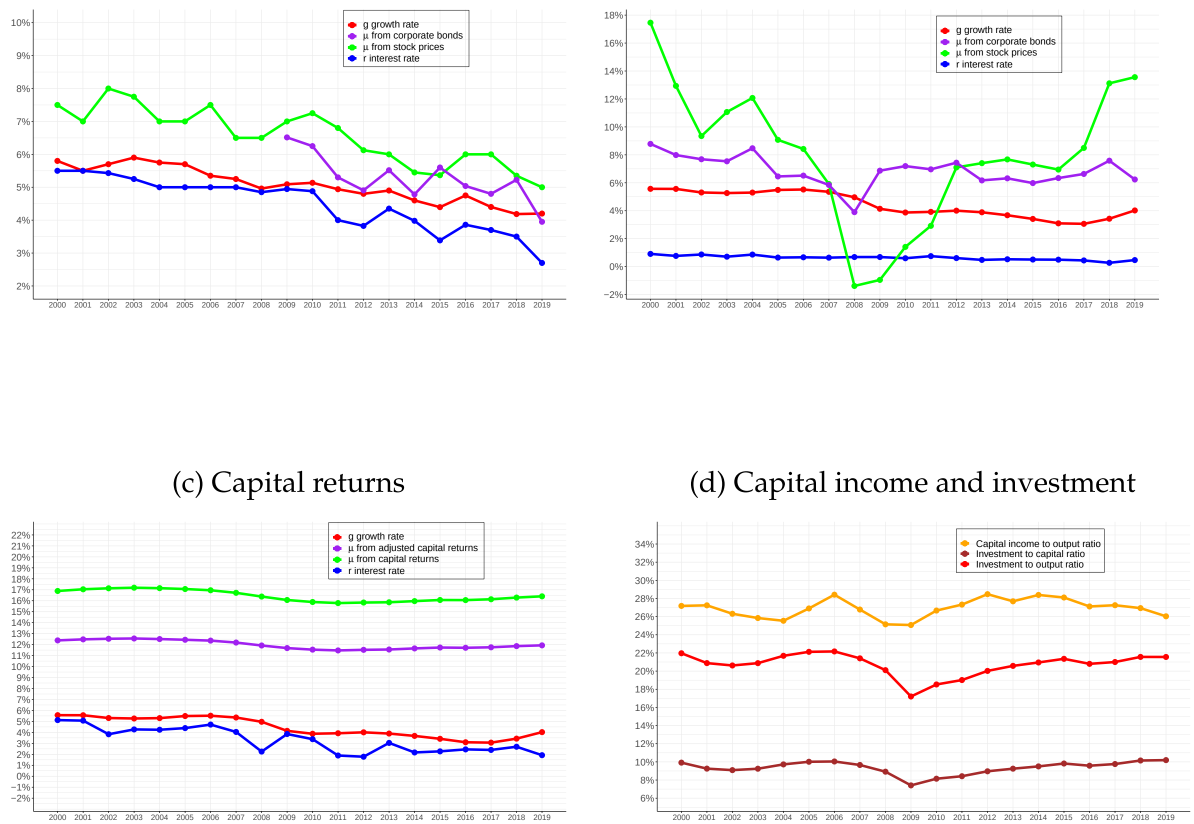
Almost every year in the past century (and maybe longer), the long-term interest rate on US government debt (r) was below the growth rate of output (g). In the past two decades, this has also become the norm for most advanced economies. When $r < g$ forever, a deficit today need not be paid with future taxes, as the debt it leads to will shrink over time as a share of output. Moreover, permanent deficits can be financed with a bubble in government debt. These two conclusions inspire proposals for running persistently large public deficits, and aiming for higher steady-state levels of debt-to-GDP. Taken to the extreme, they suggest that the sky is the limit when it comes to how much the government can spend and borrow.

Yet, at the same time that $r < g$, the US data strongly suggests that the marginal product of capital (m) has stayed relatively constant, well above the growth rate of output, so $g < m$. Panel (a) of Figure 1 shows the expected long-run values of these three rates, while panel (b) instead uses geometric averages over the past 10 years. In spite of the variability in asset returns, it is clear that $r < g < m$. Panel (c) measures the marginal product of capital using capital income, as opposed to asset prices, which even after subtracting for depreciation is higher than the growth rate of economy. Finally, $g < m$ is typically stated as a condition for the dynamic efficiency of the economy, but Abel et al. (1989) noted that a sufficient condition for dynamic efficiency is instead that capital income exceeds investment at all dates. Panel (d) plots both of these as a ratio of GDP to show the economy seems clearly dynamically efficient, as well as the investment to capital ratio, which would be a lower bound for m. The US economy appears to be dynamically efficient, and there is no noticeable downward trend in these variables.[1]

This paper starts in section 2 by showing that if $r < g < m$, then there is still a meaningful government budget constraint once future surpluses and debt are discounted by the marginal product of capital. The government can run a deficit gamble by having a one-off surge in spending and rolling over the associated debt for long enough that the eventual payment is a negligible share of national income. The present value of primary surpluses can be negative, because there is a bubble component of public debt that subtracts from the outstanding debt that must be paid. The government earns a bubble premium on the debt, which is the difference between the return that private agents can earn on the marginal unit of capital as opposed to lending to the government. If the marginal

[1]For further discussions on the measurement of r, g, m see Gomme, Ravikumar and Rupert (2011), Geerolf (2018), Barrett (2018), Mauro and Zhou (2020).

Figure 1: The US marginal product of capital, growth rate, and interest rate since 2000



(a) Expected returns

(b) Past average returns

(c) Capital returns

(d) Capital income and investment

Notes: Panel a) plots the expected returns on Treasury bonds, real GDP growth rate plus PCE inflation rate, stock returns, and returns on Baa corporate bonds, all at a 10-year horizon, according to the median respondent to the Survey of Professional Forecasters. Panel b) plots the geometric 10-year averages of Treasury 10-year bond returns, nominal GDP growth rate, r the returns on SP500 index, and returns on an index of Baa corporate bonds. Panel c) has the ay-year geometric average of a yield on a 10-year Treasury, the same output growth rate as in panel b), and a geometric 10-year average of the ratio of net value added minus labor expenditures to the corporate capital stock in the non-financial corporate sector from the Bureau of Economic Analysis' Survey of Current Business, and an adjusted return on capital that takes away 5% of GDP from capital income to account for land income, and 2/3 of proprietary income, attributed to a remuneration for labor. Panel d) plots point-in-time capital income series, now as a ratio of GDP, and the investment to capital and investment to output ratios using the BEA's data for non-financial corporate investment, capital stock and value added.

product of capital exceeds the growth rate of the economy by enough, then the government can run a perpetual deficit, that is paid for by the bubble premium revenues. A higher $m - g$ lowers the present value of the future spending, while a higher $g - r$ raises the value of the bubble, both allowing for more spending. There is an upper bound on the size of the perpetual spending: the public deficit as a ratio of the privately-held assets in the economy can at most be $g - r$. For the US this upper bound is somewhere in the range of 2.5-6% of GDP.

Since in a dynamically efficient economy, the public debt constraint depends on all of r, g, m, one needs a model to simultaneously pin down all three respecting the observed ranking between them. Section 3 offers one such model. Infinitely-lived households invest in linear technologies, but financial market are incomplete because the most productive entrepreneurs cannot borrow as much as they would like to invest in their technology. Least productive and risky firms are in business, creating a misallocation of resources that endogenously drives the wedge between the marginal product of capital and the growth rate of the economy. Government bonds provide two services to its holders: safety from production risk, and liquidity in the form of a store of value that complements private credit. These match two of the most commonly estimated reasons for the $r - g$ differences that we observe in the data, and the model offers a simple way to model them and study how they are affected by policy.

Section 4 looks at the other side of the lessons from section 2: now it is the public spending that is kept fixed, and the question that is investigated is how does raising that spending affect the endogenous growth rate and interest rate. Higher public spending as a ratio of the debt raises the interest rate on public debt. The growth rate may rise or fall, but inequality unambiguously rises. The share of private assets employed productively rises, while the share saved in public debt falls. Yet, this process can only be taken so far. There is an upper bound for the permanent spending, which, if crossed, leads to debt becoming a Ponzi scheme that private agents no longer want to hold, so the bubble pops and the value of the public debt falls to zero.

These persistent spending policies interact with other government policies. Section 5 studies this interaction with four such policies: redistribution, taxation, monetary policy, and financial regulation. There are spillovers that create conflicts between different policymakers. The more surprising is the prediction that tax-transfer systems that reduce inequality will lower the amount of other public spending that the government can make. When inequality is lower, the differences $g - r$ and $m - g$ that created the bubble in government debt become smaller, so the government budget constraint is tighter.

Higher taxes on the one hand raises revenue, but on the other hand they reduce not only real activity but also private credit in the economy. Therefore, they shrink the bubble in public debt even as they raise the present value of fiscal surpluses. The upper bound on the public deficit so that the bubble exists may be larger or smaller depending on which effect is larger. But, for a fixed interest rate, public spending must fall if taxes rise. Or, from the other direction, tax cuts allow spending to increase: they pay for themselves.

Turning to monetary policy, expected inflation has no effect on the government budget constraint. Inflation volatility lowers the real value of the nominal debt whenever there is a positive shock to inflation. However, these occasional ex post instances of debasing the debt come with an ex ante increase in the inflation risk premia, which raises r and reduces the difference $r - g$. Attempts to inflate away the debt when bond holders are forward-looking reduce the bubble value of the debt, and so tighten the budget constraint of the government. There is no conflict in the mandates of the central bank and the fiscal authority, since delivering stable inflation is what creates the most fiscal space to raise public spending.

Finally, financial repression succeeds at allowing for persistently higher spending by creating a repression premium on the public debt, even if they lower the bubble premium. However, forcing lenders in the economy to hold part of their assets in government bonds that pay below-market rates, comes at a cost in consumption and output growth.

Section 6 concludes.

Link to the literature. The model builds on Reis (2013) and Aoki, Benigno and Kiyotaki (2010), by generating misallocation within a sector because the more productive firms are bound to borrow less than a fraction of their future revenue. Those papers studied the effect of large swings in capital flows from abroad on the misallocation to explain productivity slumps like the one experienced in Southern Europe after joining the euro. This paper instead considers a closed economy, introduces uncertainty, and focuses on bubbles and public debt, which were not studied in those papers.

A long literature has studied the conditions for $r < g$ and for there to be a bubble in public debt when there are overlapping generations (Diamond, 1965, Tirole, 1985). This paper instead models a bubble in government debt when households live forever. Its existence relies on incomplete markets as in Santos and Woodford (1997), Kocherlakota (2008), Hellwig and Lorenzoni (2009), but those papers study endowment economies, whereas this paper has a production economy so that it can have a marginal product of capital. The model is closer to the production economies in Farhi and Tirole (2012), Aoki, Nakajima and Nikolov (2014), Hirano, Inaba and Yanagawa (2015), Martin and Ventura (2018), but these do not include government debt, which is the focus of this paper. Brunnermeier, Merkel and Sannikov (2020) also study the impact of market incompleteness and bubbles on the government budget constraint, but their focus is on determining inflation, which here is taken as given. The focus on the sustainability of public debt is shared with Bassetto and Cui (2018), Mehrotra and Sergeyev (2020), but the interaction with other policies is novel. More generally, the literature on bubbles and the public debt often focuses on whether bubbles exist and how they affect welfare, whereas this paper instead studies how the size of the bubble in public debt varies, and how this affects fiscal space through the government budget constraint.

On the public debt, Ball, Elmendorf and Mankiw (1998), Blanchard (2019) argued that given $r < g$, government could either run prolonged deficits with minimal impact on fiscal space, or aim for a larger steady state debt-to-GDP. This paper re-examines these conclusions when $g < m$, and investigates how fiscal, monetary, and financial polices affect the ability to undertake deficit gambles or carry larger debt. The fiscal implications of $r < g < m$ are also discussed by Barro (2020), and the gap is also partly justified by one of the two features of the model in this paper, the safety of government debt. But, there is no liquidity benefit to the debt, and Ricardian equivalence holds in a representative agent framework with aggregate shocks, so this paper can investigate the interaction with other policies. Finally, in this paper, the forces driving $r < g < m$ are the safety provided by government debt and its ability to provide an alternative store of value to incomplete private markets. These would appear as risk premia, consistent with the findings in Caballero, Farhi and Gourinchas (2017), Farhi and Gourio (2018), Mark, Mojon and Veldes (2020) that a rise in risk premia explains the bulk of the fall in r and increasing gap $m - r$ in the last two decades. Farhi and Gourio (2018), Eggertsson, Robbins and Wold (2020) further argued that measured m using capital income may exceed the actual one because of economic profits, and explain the rising gap between the two. I leave for future research to study the consequences of possibly rising market power on the public debt constraint.

Finally, on the government debt constraint, Jiang et al. (2019) also propose discounting the government budget constraint using a measure of marginal capital, but with a very different purpose than this paper. Hilscher, Raviv and Reis (2014) discuss the empirical inability to inflate the public debt, whereas this paper discusses its potential ineffectiveness.

Reinhart and Sbrancia (2015) discuss the history of financial repression in the context of fiscal crises.

2 The government budget constraint

Government budgets are not easy reads, as borrowing comes through multiple instruments with different payment profiles and maturities, and spending and tax revenues depends on many other variables. There are multiple r, g, and m's that affect the constraint on how much the government can borrow through varied channels. To make the points in this paper, it suffices to simplify in two ways. To start, I assume that the total flow of resources consumed by the government on net is exogenous. I denote it by s_t in units of output, and refer to it alternatively as (net) public spending, or as the (primary) public deficit. The question in this paper is how large this can be.

Next, I assume that there is a single government bond, of which every instant a fraction ξ expires giving its holder a principal payment of 1, while the remaining $1-\xi$ pays no coupon but survives until next period. Effectively, in the model, the government is issuing a perpetuity, but one with expected maturity $1/\xi$, so that it can match the actual behavior of governments that perpetually roll over their debt, while keeping the maturity relatively stable. If $B_t \geq 0$ are the units outstanding of this bond, then its value in output units is: $b_t = B_t v_t / p_t$ where v_t is the nominal value (or price) of the bond, and p_t is the price level.

The return on the bond is: $r_t = \xi + (1-\xi)\frac{dv_t}{v_t} - \frac{dp_t}{p_t}$ where the first term is the coupon rate (or yield), the second term is the capital gain, and the third term is the inflation loss. This return, r_t, is the first endogenous variable that the model later will determine. Even if the government can choose the maturity ξ, or even how many bonds to sell B_t, the return on the government debt is endogenous as the market price adjusts as needed to clear markets. Also, let $\bar{r}_t = (\int_0^t r_s ds)/t$ be the average return between dates 0 and t.

Combining the assumptions, the law of motion for the evolution of the public debt is:

$$db_t = s_t dt + r_t b_t dt. \tag{1}$$

The debt increases because of spending plus paying interest on the debt (the total public deficit). At the same time, output denoted by y_t, also rises at rate g_t. This growth rate is the second key endogenous variable in the model.

2.1 Four consequences for the constraints on public spending and debt

From the facts in section 1, take as given that for large enough t, we have $\bar{r}_t < \bar{g}_t < \bar{m}_t$, so on average, in the long run, the marginal product of capital exceeds the growth rate, which exceeds the return on the government debt. I will endogenize the rates in the rest of the paper, but for this section, take them as given. These two inequalities have four important consequences for how, and whether, public spending and debt are constrained.

Imagine what would happen if the government had a one-time spending splurge at date 0, s_0. Then, after t periods, the debt-to-output resulting from this "deficit gamble" is: $e^{(\bar{r}_t - \bar{g}_t)t} s_0$. Therefore, if average growth is higher than the average return on government debt, this shrinks exponentially fast to zero. The government can roll over the debt for t periods, and if t is large enough, end up having to cut spending by a negligible amount as a fraction of output in that distant future. It can easily pay for the gamble, no matter how large the splurge was, as long as it does not exceed output that period.[2] This is the first consequence of $r < g$:

Lesson 1: *One-time deficit gambles are feasible as long as the spending splurge keeps $r < g$.*

Second, imagine instead that the government has a permanent deficit, and it raises its spending, on average in the long run, at the rate of growth of output. Solving the differential equation in equation (1) forward gives rise to:

$$b_0 = \lim_{T \to \infty} \left[- \int_0^T e^{-\bar{r}_t t} s_t dt + e^{-\bar{r}_T T} b_T \right] \tag{2}$$

Now, since s_t also grows at rate $\bar{g}_t$, the limit of the first integral by itself is not well defined. Moreover, if the government pursues a Ponzi scheme, rolling over the debt forever, so that b_T also grows at the rate of output on average in the long run, then the second term is equal to infinity. Encountering these infinities, one might think that debt has no upper bound. The government could then spend whichever arbitrary amount it wants, up to the resources available in the economy, and count on running a Ponzi scheme and never pay for it. There would be no constraint on public spending and debt.

Yet, a mathematical expression may be undefined but the appropriate economic constraint still be defined. In fact, note that the flow budget constraint can be written as

[2] Of course, if the splurge was as high as output, then intertemporally smoothing consumers for whom the marginal utility of consumption approaches infinity as consumption goes to zero, would drive the interest rate to infinity, breaking the premise of the exercise. The next sections deal with the endogeneity of interest rates.

$db_t - m_t b_t dt = s_t dt + (r_t - m_t) b_t dt$, and we can solve forward now using m_t to discount the future. Because $\bar{m}_t > \bar{g}_t$ for large t, the limits are now well defined and so:

$$b_0 = -\int_0^\infty e^{-\bar{m}_t t} s_t dt + \int_0^\infty e^{-\bar{m}_t t} (m_t - r_t) b_t dt. \tag{3}$$

The first term is the present value of spending, but now using the marginal product of capital as the discount rate. It is well defined since s_t cannot persistently grow faster than the growth rate of the economy, as eventually this would have the government use more than all the resources available in the economy. The second term is the bubble component of government debt, which is again well defined, since debt also cannot grow faster than output without breaking the total resource constraint. Therefore, there is still a meaningful intertemporal budget constraint for the government.

In the neoclassical growth model, $m_t = r_t$ at all dates, so the bubble term is zero. But, if public debt earns a *bubble premium*—a positive difference between the marginal product of capital in the economy and te interest rate paid on the debt—then the government budget constraint is relaxed by the present value of these premium flows. Thus, the government can pay for the existing debt partly through a present value of surpluses, but partly also through the bubble revenue from the public debt. In the extreme, the present value of spending can even be positive in spite of outstanding debts to pay, as long as the bubble value of debt is higher than the current value of the debt. This is the second consequence of having $r < g < m$:

Lesson 2: *The present value of spending must stay below the bubble component of debt minus the current value of the debt.*

Consider now the steady state where, an instant after date 0, spending and debt grow at exactly the same rate as output growth, and all of these rates are constant. Then, equation (3) becomes:

$$b_0 = -\frac{s}{m-g} + \left(1 + \frac{g-r}{m-g}\right) b^* \tag{4}$$

where output at date 0 is normalized to 1. This shows that both differences matter, the one between r and g, as well as the one between m and g. A higher difference between g and r, all else equal, increases the bubble component of debt, while a larger difference between m and g lowers the present value of future spending. Both contribute in these separate ways to lower the right-hand side of equation (4), thus loosening the government budget constraint and allowing for more spending.

In the limit, when $r \to g$, the present value of public spending is exactly equal to the bubble debt minus the current debt. As the interest rate falls below g, the bubble exceeds current debt, and the government can run a deficit forever, financed by the flow of bubble premia. If $r - g$ falls because of a fall in r, then the budget constraint is looser, but if the cause is instead a rise in g, then the budget constraint may actually tighten because the present value of spending may rise by more than the value of the bubble

In the other direction, in the limit when $g \to m$ and the economy approaches dynamic inefficiency, the constraint is again undefined as the present value of spending approaches minus infinity, but the bubble approaches infinity. But, for a higher $m - g$, the present value of spending falls, while at the same time the present value of the bubble falls as well. If structural changes in the economy raise the marginal product of capital, then this will loosen the budget constraint of the government, and allow for higher permanent spending if $b^* > b_0$. Since $r < g$ and deficits forever are a sufficient condition for this to happen, it then follows that a higher difference between the marginal product of capital and the growth rate of the economy discounts the future spending at a higher rate, and so allows the government to spend more. This is the third lesson that applies when in steady state $r < g < m$.

Lesson 3: *If the marginal product of capital exceeds the growth rate by enough, then the government can run a perpetual deficit, which is separately increasing with both the difference between the marginal product of capital and the growth rate, or the difference between the growth rate and the interest rate.*

Finally, consider the condition where initial and steady-state debt are the same: $b = b_0 = b^*$. Then, either from equation (3), or even right away from equation (1):

$$b = \max \left\{ \frac{s}{g - r}, 0 \right\}. \tag{5}$$

The second condition makes explicit the fact that the value of the government debt has to be non-negative (the private sector cannot issue it). Recalling that the value of the debt if equal to the debt outstanding times is price, a finding that there is no equilibrium with positive b translates into the price of the debt being zero, as the private sector refuses to hold this Ponzi scheme.

The condition $r < g$ is then necessary for the government to have a perpetual deficit and run a Ponzi scheme. Moreover, for a fixed amount of spending, the size of the debt is just equal to the ratio of these deficits to the gap between the interest rate and the growth rate. Since the public debt cannot exceed the assets in the economy, this also puts an upper bound on the spending the government can do:

Lesson 4: *The maximum public deficit as a ratio of assets in the economy is given by $g - r$.*

The data in figure 1 suggests a $g - r$ difference of about 2% in 2019. At the same time, according to the Bureau of Economic Analysis, at the end of 2019, the capital stock was 2.1 times GDP, the net international investment position was -0.5 of GDP, and the privately-held public debt plus debt of the Federal Reserve was 0.8 of GDP, for total assets of 2.4 times GDP. Alternatively, using the Federal Reserve's Financial Accounts of the United States to measure private non-financial assets gives total assets of 3.5 times of GDP.[3] Therefore, the upper bound on persistent public spending is $4.8 - 7.0\%$ of GDP. Since the primary public deficit in 2009-19 was on average 4.8% of GDP, this suggests there is little in terms of extra spending that is possible if these deficits are to persist forever. Most, or all of the bubble has already been used. With the historically smaller difference of 1%, ceteris paribus, the US would have to cut its primary deficit from the expected 2020 value of 16.7% by 13.2 to 14.3 percentage points of GDP in order to respect the budget constraint. The bubble made possible by $r < g$ allows for some extra spending, but far from fiscal profligacy.

At the same time, r, g, and m would surely change in response to permanent changes in the public deficit. Having understood how these three variables affect the fiscal possibilities of the government, one needs a model that endogenizes them to make further progress on how policy affects the public debt constraint.

3 A model where safety and liquidity are scarce

I consider an incomplete-markets version of a standard AK economy. There are no transition dynamics and no aggregate risk, so the analysis is kept simple by only having to solve for steady states. But, at the same time, the incompleteness of markets generates inequality in opportunities and outcomes that interact with policy to determine the growth rate and the interest rate.

[3] The difference between the two numbers is mostly about whether to include land. Arguably, the private sector cannot convert land into government debt, so the lower number is more accurate.

3.1 The economy

Aside from the government, already presented in the previous section, the economy has a representative competitive firm and a continuum of households indexed by i.

3.1.1 The firm

The neoclassical firm maximizes profits by choosing how much capital to hire from each agent k_t^i and paying them r_t^i:

$$\max\left\{A\int_0^1 q_t^i dt k_t^i di - \int_0^1 (r_t^i dt + d\delta_t^i) k_t^i di\right\}. \tag{6}$$

Total factor productivity is a constant A, but the capital of each individual must be adjusted for its quality q_t^i. In turn, the use of each of the capital stocks leads to a depreciation of $d\delta_t^i$ in an instant of time.

There are only two types of capital in the population, given by the following expressions:

$$q_t^i = \begin{cases} 1 & \text{if } i \in E \\ \eta & \text{if } i \in F \end{cases} \quad \text{and} \quad d\delta_t^i = \begin{cases} \delta dt & \text{if } i \in E \\ \eta\delta dt + \sigma dz_t^i & \text{if } i \in F \end{cases} \tag{7}$$

If the household is in the E group, for entrepreneurs, then quality is high (normalized to 1), and depreciation happens at a constant rate δ. This is just like in the neoclassical growth model.

The remaining share of households are financiers in group F, in that the quality of their capital is worse and the depreciation is volatile. As soon as the firm uses this capital, its effective units fall by $\eta < 1$, and these effective units depreciate by not just δ but also by an extra shock represented by a zero-mean Wiener process dz_t^i that has variance σ^2. The depreciation shocks are idiosyncratic, so that by a law of large numbers they average to 0 in the population: $\int dz_t^i = 0$. There is no aggregate risk. I call these households financiers because, in a first-best world, they would all just lend their assets to the entrepreneurs, who can generate high-quality capital with investment and obtain higher and safer returns.

The forces of competition lead this firm to earn zero profits, and profit maximization make it pay each capital its marginal product, so:

$$r_t^e = A - \delta \equiv m \quad \text{and} \quad r_t^f dt = \eta(A-\delta) - \sigma dz_t^i \tag{8}$$

3.1.2 The households

Households live forever, discounting the future at rate ρ, due to impatience and selfishness towards their offspring. As usual I assume that $m > \rho$, and moreover that $\eta m - \sigma^2 > \rho$, so that all households would like to save. They obtain utility from consumption c_t^i and from the government services that the public spending provides s_t. I assume that the utility function is separable in these two sources of well-being so that, regardless of how important public services are, I can leave them out of the model as the flow of utility from s_t has no positive effect in the equilibrium on the model.

Household assets a_t^i can be used to buy government bonds, b_t^i, invest in capital k_t^i, and lend to other households l_t^i. The return on this last option is given by the interest rate r_t^l. Households cannot short public debt, or invest negative amounts in capital, but they can borrow. However, they face a borrowing constraint in that their repayment of debt cannot exceed a fraction $\gamma < 1$ of the returns from their capital. As usual, this is justified by the borrower being able to abscond with all assets but for this share of the capital stock before it is time to pay the lender. Now, because borrowing and lending between households is risk-free, but the return on financier's capital is risky, in the worst-case scenario their capital stock depreciates to zero, and they cannot pay any of the loan back. Therefore, their natural debt limit is to not be able to borrow at all. Going forward, I refer to γ as the level of financial development of the economy, since the larger it is, the larger is the private debt market.

Combining all the ingredients, each household solves the following dynamic problem:

$$\max_{\{c_t^i, b_t^i, l_t^i, k_t^i\}} \mathbb{E}_0 \left[\int_0^\infty e^{-\rho t} \log c_t^i dt \right]$$

$$\begin{aligned}
\text{subject to: } a_t^i &= b_t^i + l_t^i + k_t^i \\
da_t^i &= (r_t b_t^i + r_t^l l_t^i + r^i k_t^i - c_t)dt \\
b_t^i &\geq 0, k_t^i \geq 0 \\
-r_t^l l_t^i &\leq \gamma r_t^i k_t^i \;\; \text{if } \; i \in E \\
l_t^i &\geq 0 \;\; \text{if } \; i \in F
\end{aligned}$$

while taking initial assets a_0^i and the returns on investment as given.

A key simplification of the model is that every period t a randomly drawn fraction α of the households becomes an entrepreneur, while $1-\alpha$ becomes a financier. These draws are iid, so even though the model generates income and wealth inequality, there are effectively only two groups of households in the population at the start of any given point of time, with all within a group making the same choices and ending up with the same income and wealth but for the depreciation shocks.

3.1.3 Market clearing and equilibrium

The economy is closed, so the market clearing conditions for the two assets are:

$$\int_0^1 l_t^i di = 0 \quad \text{and} \quad \int_0^1 b_t^i di = b_t. \tag{9}$$

Total assets are denoted by $a_t = \int a_t^i di$, and the iid assumption on types implies that entrepreneurs at the start of a period have wealth $a_t^E = \alpha a_t$, while financiers have $a_t^F = (1-\alpha)a_t$. An equilibrium is a situation where the firm and households behave optimally, the government budget constraint holds, and the markets clear, given an initial level of assets a_0 and a fiscal deficit s.[4]

Because this is an AK-type economy, when there is some lending between households, aggregate consumption, assets, capital and output, will all grow at the same rate. This growth rate, g, will be constant over time, as will be the safe rate r (and all other interest rates). Moreover, since all variables grow at the same rate, they all can be expressed relative to the initial level of assets a_0.

All that remains to specify is the exogenous value (or rule) for initial government spending s. If $r > g$, then no s except 0 would be consistent with an equilibrium, since there are no sources of public revenue.[5] But, as we will see, this economy can deliver $r > g$ and, following the exposition in the previous section, therefore support permanent deficits. I assume that policy chooses a ratio of spending to debt, or s/b since, from equation (5), this is equal to $g-r$. Therefore, the comparative statics can be stated as raising or lowering $r-g$ (which we observe in the data anyway).[6]

[4] Recall that in the previous section, we already assumed that s will grow over time at the rate g, so only the initial s is exogenous to the model.

[5] Adding lump-sum taxes would make no difference, and simply lead to a reinterpretation of s as spending net of those taxes.

[6] In other words, the point of the model is not to ask whether $r < g$; it will, and instead the focus is on

A solution of the model is then a growth rate g, an interest rate r, and the size of the public debt as a ratio of total assets in the economy $\beta \equiv b/a_0$, as a function of exogenous public spending s/b. An equilibrium may not exist for $s > 0$, and the model imposes the resource constraint: $\beta \in [0,1]$. The existence of an equilibrium that satisfies this constraint translates into a condition that spending is not too large: $s/b \leq S$. So, the upper bound S must also be solved for.

3.2 The roles of public debt

In the model, there are no markets to trade the idiosyncratic risk that financiers bear if they invest in capital, as a result of depreciation. Markets are incomplete as financiers cannot insure against the chance that their capital is wiped out. The public debt is safe for the households, since its returns are uncorrelated with the returns on individual capital. Public debt therefore provides *safety*, which the agents demand.

Moreover, public debt provides an alternative to store wealth over periods for financiers if the constraint on private lending is too tight. Financiers are aware that they can become entrepreneurs next instant, or at any point in the future, and want to store value for when they get a high-quality investment opportunity. Public debt therefore also provides outside *liquidity*, complementing the inside liquidity from private lending, which the agents value.

The model above is a simple vehicle to capture these two important roles of public debt. With it, one can investigate whether persistent spending that tries to take advantage of the bubble premium is consistent with optimal behavior and markets clearing. If it is, then the size of this spending will endogenously determine r and g. Policies and public debt will change the relative strengths of the safety and liquidity effects, and so may move the two key differences, $m - g$ and $r - g$ in either the same or opposite directions, with the fiscal consequences already discussed in section 2.

3.3 Equilibrium benchmarks without government debt

Before doing so, it is useful to set up two benchmarks at two polar extremes of financial development where public debt has no effect on the equilibrium growth and interest rate.

The first is when financial development is very high: $\gamma > 1 - \alpha$. In that case, the financiers can lend all of their assets to the entrepreneurs to invest in their high-quality

the consequences of this for the overall level of g, r, and the size of the public debt.

assets. The financiers will wish to lend, since their technology is inferior in both expected return and risk, and the entrepreneurs will strictly wish to borrow as much as they can as long as their investment returns exceed the lending rate. In equilibrium, for the lending market to clear, it must be that $r_t^l = m$. Both agents choose to consume a share ρ of their assets and save the rest, so that assets and consumption for all households grow at rate $g = m - \rho$. In turn, since lending is risk-free, the rate on the public debt is also $r = m$. I refer to this equilibrium as the first best:

$$m = r^{\text{first-best}} > m - \rho = g^{\text{first-best}} \tag{10}$$

This is just a textbook neoclassical growth model with an AK production function. The first welfare theorem holds so the economy is dynamically efficient and $m > g$. The model is unable to generate $r < g$ as we see in the data. Therefore, there is no bubble premium, and the public debt must equal the present value of surpluses. If the governments tries to spend forever $s > 0$, then the equilibrium requires that $b = 0$, so public debt is worthless, and the government fails to do any spending.

At the other extreme, assume an economy with no public debt ($b_t^i = 0$), and where no private lending or borrowing is allowed ($l_t^i = 0$). The entrepreneurs still have their consumption and assets grow at rate $m - \rho$, but now they can only invest the assets they start each period with, αa_t. The financiers instead can only invest in their risky-low quality capital. On aggregate, their consumption and assets grow at rate: $\eta m - \rho + \sigma^2$. On the one hand, this is lower than that of entrepreneurs because their technology is less productive on average, $\eta < 1$. On the other hand, it is higher because the risk $\sigma^2 > 0$ induces precautionary savings, so the agents consume less and save more.

Adding over the two groups, weighted by their asset shares, gives the growth rate of the economy. As for the safe rate, if a safe bond were introduced for entrepreneurs, its shadow safe rate would be m, while if it was financiers, the shadow safe rate would be instead: ηm. I take the wealth-weighted average of these to end up with the following rates in autarky:

$$m - g^{\text{autarky}} = \rho + (1-\alpha)(1-\eta)m - (1-\alpha)\sigma^2 \tag{11}$$

$$g^{\text{autarky}} - r^{\text{autarky}} = -\rho + (1-\alpha)\sigma^2 \tag{12}$$

In this autarkic case, the gap between the growth rate and the interest rate is higher than in the first best: $g^{\text{autarky}} - r^{\text{autarky}} > g^{\text{firstbest}} - r^{\text{firstbest}}$. This is because the financiers, unable to lend for a safe return, have a large desire for safety that drives down the safe rate. If $(1-\alpha)\sigma^2 > \rho$, the autarkic economy can generate the pattern that we observe in the data where the interest rate is lower than the growth rate of the economy. This is because as long as there is enough uncertainty, the desire for safety is strong enough to push the interest rates below the growth rate.

This idiosyncratic risk also reduces the gap between the marginal product of capital and the growth rate of the economy. This is because risk makes financiers save more and so grow faster (while being worse off in utility terms): the safety effect. The other force in the model pushing to raise this gap is the lack of liquidity. The absence of outside liquidity means that financiers can no longer store any value over time at the high rate of return that they could by lending to entrepreneurs, so the lower is η the less the growth rate. If the liquidity effect is stronger than the safety effect, which happens if $(1-\eta)m > \sigma^2$, then $m - g^{\text{autarky}} > m - g^{\text{first-best}}$.

4 The effects of public debt on equilibrium

Between these two extremes of financial development, when $0 < \gamma < 1-\alpha$, different economies will have different configurations for the two key wedges. It turns out that the economy can be in one of two situations: if $\gamma \in (0, \gamma^*)$, which I call the *financially under-developed economy*, and if $\gamma \in (\gamma^*, 1-\alpha)$, which I call the *financially developed economy*, even though it is not quite at the first best. The threshold γ^* is derived in the appendix as a function of the fundamental parameters.

More importantly, either in one range or the other, public debt will provide some outside liquidity and some safety, and so affect the equilibrium of the two rate differences. The analysis proceeds by considering each case in turn.

4.1 A financially developed economy

In a financially developed economy, no financiers choose to invest in their inferior capital stock. Rather, there is enough private credit that they prefer to lend their funds to the entrepreneurs, earning r^l, which is close enough to the return in the superior technology m. But then, there is no idiosyncratic risk borne by anyone in the economy, and so the public debt will only play the role of providing liquidity.

4.1.1 Solving the model

The entrepreneurs borrow as much as they can, and the financiers are happy to provide the funds up to the borrowing limit. As a result, the entrepreneurs' assets (and consumption) grow at the rate:

$$\frac{\dot{a}_t^E}{a_t^E} = \frac{(1-\gamma)mr}{r-\gamma m} - \rho \tag{13}$$

and the entrepreneurs hold no government bonds. As for the financiers, since they must hold the government bonds for that market to clear, then $r = r^l$. The growth rate of the financiers' assets and consumption is:

$$\frac{\dot{a}_t^F}{a_t^F} = r - \rho \tag{14}$$

The growth rate of the economy is the weighted average of these two rates, with weights α and $1-\alpha$, respectively. In turn, the budget constraint of the government imposed that the growth rate is equal to $r + s/b$. Replacing out g, and rearranging gives the equilibrium condition:

$$\frac{\alpha(m-r)}{1-\frac{\gamma m}{r}} = \rho + \frac{s}{b} \tag{15}$$

Since the left-hand side is continuous and monotonic in r, this pins down the unique r solution of the model.

Total assets are split between investment in the superior technology and holdings of public debt. Entrepreneurs' assets equal $k^E - \gamma r^k k^E / r$, since they borrow to invest in capital abode their assets. Using the equilibrium rental rate of capital, the fact that a share α of household assets are in the hands of entrepreneurs, and the definition of the debt-to-asset ratio β, gives the other equilibrium condition:

$$\beta = 1 - \frac{\alpha r}{r - \gamma m} \tag{16}$$

This uniquely solves for the size of the public debt given a solution for r from the previous equation.[7]

[7] The debt can never be larger than $1 - \alpha/(1-\gamma)$, otherwise the economy stops being dynamically efficient.

4.1.2 The effect of more public spending

Lesson 4 noted that the maximum persistent spending is given by $g - r$. In this model, if the government raises its spending, in equilibrium $g - r$ will rise, making this possible. However, this comes with a side consequence and a limitation. The former is that r and g will individually change, so that the higher spending will affect welfare and growth. The latter is that there is an upper bound S up to which this is feasible, and if exceed leads the bubble on public debt to pop, driving its value to zero

From equation (15), more permanent spending by the government (higher s/b) lowers the interest rate. The left hand side falls with r, while the right hand side does not depend on r but rises with s/b. Therefore, by the implicit function theorem, r depends negatively on government spending. The intuition is that when the government wants to spend more, paying for it requires that there is a larger bubble component in the public debt. Since the marginal product of capital is fixed, the bubble premium comes about through a lower interest rate on the debt.

At the same time, households will want to hold less public debt as a share of their assets, since it pays a lower return. We can see this because the right-hand side of equation (16) is increasing in r.

Since $g - r$ increased, the entrepreneur's borrowing constraint was relaxed, so they are able to borrow more. Depending on parameters, the growth rate g may rise or fall. However, because of the rise in $g - r$, for sure there is greater inequality in income (and so consumption and asset growth) between entrepreneurs and financiers. This is because the entrepreneurs are borrowing more and on cheaper terms from the financiers. Larger government spending comes with more growth but also greater income inequality, as it shifts some capital form the financiers to the entrepreneurs.

Finally, the upper bound on the size of the permanent deficits for there to be an equilibrium where the public debt has positive value is:

$$S = (m - \rho)\left(1 - \frac{\gamma}{1 - \alpha}\right) - \rho. \tag{17}$$

This confirms the previous result that, if the economy is sufficiently developed, so $\gamma > 1 - \alpha$, then the government cannot run permanent deficits, because the economy is in the first best where $r > g$. As the economy is less financially developed, then public spending can be higher.

In fact, the lower is the share of assets in the hands of entrepreneurs—either because the lower is financial development (lower γ), or because there are fewer entrepreneurs or they had fewer assets to born with (lower α)—the more public spending can be sustained by rolling over the public debt. Intuitively, public debt provides a store of value for the households who do not have access to good investment opportunities. The inability of the private economy to allocate resources to its most productive members provides a demand for public debt. Households can save in public debt the expectation that they might become entrepreneurs in the future. This liquidity service gives rise to a bubble premium, and so the government can afford to spend more.

4.2 A financially under-developed economy

If $\gamma/(1-\alpha)$ is too low, the equilibrium in the economy shifts. At this point of financial underdevelopment, the private credit market is too small and financiers end up investing some of their assets in the low-quality capital stock. The economy will exhibit a misallocation of capital, as aggregate TFP is lower and its cross-section dispersion is higher.

4.2.1 Solving the model

The financier will invest:

$$\frac{k^F}{a^F} = \frac{\eta m - r}{\sigma^2}. \tag{18}$$

This is the standard result that the share of assets allocated to the risky investment is equal to its Sharpe ratio. The appendix derives the growth rate of assets aggregated across financiers, which is:

$$\frac{\dot{a}_t^F}{a_t^F} = r - \rho + \sigma^2 \left(\frac{k^F}{a^F}\right)^2 \tag{19}$$

Relative to the developed economy, the presence of risk makes the financiers consumer less due to precautionary savings, and so their assets grow at a faster rate.

The entrepreneurs' behavior is the same as in the developed economy. Aggregating over all agents to get the growth rate of the economy, substituting in the government budget constraint, and rearranging, just as in the previous section delivers the equilibrium condition:

$$\frac{\alpha(m-r)}{1-\frac{\gamma m}{r}} + \frac{\sigma^2}{1-\alpha}\left[1-\beta-\frac{\alpha r}{r-\gamma m}\right]^2 = \rho + \frac{s}{b} \tag{20}$$

Relative to the developed economy, there is a new term on the right-hand side. This captures the demand for safety by the households. All else equal, it generates lower interest rates.

The second equilibrium condition, following the same steps as in the developed economy, is:

$$\beta = 1 - \frac{\alpha r}{r - \gamma m} - \frac{(1-\alpha)(\eta m - r)}{\sigma^2}. \tag{21}$$

The last term is new, and again captures the demand for safety. The larger is the variance of the returns to investing in the capital stock, the larger is the share of assets that the households will want to save in government bonds.

Solving for equilibrium now requires solving these two equations jointly for r and β. The appendix does so, and proves that there is a unique solution.

4.2.2 A special case and the effects of public spending

A special case helps to develop intuition. Imagine that $\alpha = 0$, so there are only financiers in the world, and no good-quality technology. In this case, there is no role for the public debt in allowing households to transfer value into the future when they might become an entrepreneur. Only the role for public debt as providing safety remains, so this special case allows us to isolate and study it.

Rearranging the two equations above, the solution of the model in the $\alpha = 0$ case is:

$$r = \eta m - \sigma \sqrt{\frac{s}{b} + \rho} \tag{22}$$

$$\beta = 1 - \frac{1}{\sigma} \sqrt{\frac{s}{b} + \rho} \tag{23}$$

Note right away that an increase in uncertainty (σ) raises the desire for precautionary savings. So, it raises the holdings of government debt β, while pushing down the interest rate r.

An increase in permanent spending as a ratio of debt, as before, raises $g - r$. The expression show that both r and β fall, just as happened in the developed economy, where there was no demand for safety. The intuition now is that for $g - r$ to increase, then households must choose to invest more in the risky technology than in the government bonds. Because this increases overall risk, then the safety of government debt is more valuable, its bubble premium rises, and so more persistent spending is possible. Also as before, both investment and inequality are now higher, since the lower safe interest rates induce households to invest more in their risky technologies, which have dispersed returns. Finally, whether the growth rate rises with government spending, again depends on parameters: it will if $s/b > \sigma^2/4 - \rho$, that is once spending is high enough.

The upper bound on spending so that the government can spend forever is now:

$$S = \sigma^2 - \rho. \tag{24}$$

There has to be enough risk in the economy to drive r sufficiently down and create a bubble premium in the public debt. If so, then the government can run a permanent deficit up to the variance of investment returns minus the subjective discount rate. Again, as in the developed economy, the upper bound S is higher the less developed the financial markets are, here in the sense of higher idiosyncratic investment risk that cannot be diversified away.

4.3 Conclusion from endogenizing rates

The two motives for why $r < g$ in the model— the uses of public debt as a store of value and as a safe harbor—complement each other. The conclusions from both models on what happens after an increase in public spending were qualitatively the same. I collect them in the following lesson:

Lesson 5: *Higher public spending as a ratio of debt lowers the safe interest rate, may raise or lower the growth rate, raises the share of assets invested in production, and raises inequality. The increase in the bubble premium supports the higher spending, but there is an upper bound on far it can go, and this bound becomes tighter if financial markets become more developed.*

From now onwards, I will focus on the developed economy, since analyzing it is slightly simpler, and the results are qualitatively similar.

5 Policy tradeoffs

Redistributive, fiscal, monetary, and financial regulation policies affect the equilibrium growth rate and interest rate in the economy, interacting with the amount of government spending. Therefore, they affect the bubble premium on the government debt, and so the ability to run perpetual deficits and their size. Insofar as the policies are chosen by a different policymaker than the one choosing public spending, conflicts will arise. This section studies these effects, and the trade-offs they give rise to.

In particular, I ask two questions: First, if government spending responds to a change in policy by keeping the interest rate fixed, does the policy lead permanent spending s/b to rise or fall? Keeping the interest rate fixed allows me to focus on the direct effects of these policies, by removing the indirect effects of the policies on income or inequality through the general equilibrium feedbacks discussed in the previous sections. Second, how does the upper bound on permanent spending S change with the policy?

5.1 Redistributive policy: lowering inequality versus raising public spending

Entrepreneurs in this economy have higher income and consumption than do financiers. If the economy is relatively financially underdeveloped ($\gamma/(1-\alpha)$ is small) , this inequality will be starker. Moreover, all the households are ex ante identical, so utilitarian social planners will be tempted to address this inequality by taxing the entrepreneurs and transferring funds to the financiers. However, redistribution usually comes with distortions to incentives. The attempt to provide social insurance through taxes and transfers can therefore have spillover effects on the whole economy, including on the government itself in its ability to run persistent deficits.

For concreteness, I assume that there is only a progressive transfer program based on income available. Since there are only two income groups in the population, the income subsidy to the poor financiers is a fraction χ of their income. To focus on redistribution alone, the total transfers are paid for by taxing entrepreneurs's income at a constant rate to as to balance the redistribution budget. The higher is χ the larger is the size of the redistribution program. The appendix proves the following result:

Lesson 6: *A larger redistribution program lowers the persistent public spending that is feasible, if the government keeps the interest rate unchanged. Starting from a zero-redistribution economy, raising redistribution also lowers the upper bound on spending that the government can sustain without driving the value of the debt to zero.*

The intuition is that more redistribution implies that the financiers are even more willing to lend to entrepreneurs. They now have higher post-transfer assets, and they want to save them for when an entrepreneurial opportunity arises. This causes the interest rate to fall. Using the results from the previous section, in order to keep the interest rate fixed, the government must cut on spending.

In the model, the poor have few investment opportunities, so they want to lend them out. The financial friction, unfortunately, preventing them from lending them out to entrepreneurs that could put them to good use. It traps their assets in their poor owners, which are forced to them to unproductive uses, like government debt or inferior technologies. Redistribution, by raising the assets of the poor, pushes the financial market to do more work. Therefore, it has the same effect as making the economy more financially developed, which from the intuition in the previous section, lowers the sustainable maximum amount of persistent spending.

A policymaker that is focused on inequality and approves a large transfer program will then constraint the ability of a different policymaker that is focused instead on the provision of public services or infrastructure. A conflict will arise, especially in a political system where parties alternate in power and have different preferences for inequality and vis-a-vis public spending, for instance in defense. It is well known, empirically and theoretically, that this may lead to over-spending. Since the upper bound on spending is also lower, there is a heightened risk that public excesses in either redistribution or spending end up popping a bubble, and causing a debt crisis.

5.2 Fiscal policy: tax cuts that pay for themselves?

So far, I have considered the effect of public spending, but not that of public revenues. The appendix changes the model to have the government levy a proportional income tax that is the same for all agents. It then proves that:

Lesson 7: *An income tax cut raises the persistent public spending that is feasible if the government keeps the interest rate unchanged. It may raise or lower the upper bound on spending that the government can sustain without driving the value of the debt to zero.*

A cut in the tax rate lowers fiscal revenues and raises the primary deficit. All else equal, the direct effect of it is to reduce the amount of spending that the government can undertake. But, this policy also raises the returns to investment, income and tax revenues. This is the well-known incentive effect of tax cuts on income. More interesting, in the model, the increase in the income of entrepreneurs raises their ability to borrow in the private credit. Investment in high-quality capital rises and so does growth. This provides a new source of extra revenue for the government. Combined, the government finds itself with more resources available to spend after the tax cut. Persistent deficits can rise forever.

However, because the tax cut allowed for more private credit, it was akin to an increase in the development of the financial market. This will tend to lower the maximum size of the persistent deficits, as discussed in the previous section. Whether this effect, or the extra revenues, raise or lower S will depend.

The question of whether tax cuts ever pay for themselves is a classic one in economics. The empirical debate revolves around measuring the tax multiplier, the extent to which out and ultimately the tax base rises after a fall in the tax rate. The perspective offered in the above lesson is quite different. First, because it suggests that deficit-financed tax cuts, by increasing the public debt, raise a source of revenue for the government. Second, because it suggests measuring how $g - r$ responds to a tax cut, which would combine estimates of multipliers with estimates of direct crowding-in effects of tax changes on interest rates, but where the elasticity of investment to interest rates is irrelevant. These are intriguing cues for future research to pursue.

5.3 Monetary policy: inflating the debt or deflating the bubble?

Assume that inflation is positive and stochastic:

$$dp_t/p_t = \pi_t dt + \sigma_\pi dz_t^\pi, \tag{25}$$

where π_t is the expected inflation rate, and dz_t^π are aggregate shocks to inflation, uncorrelated with the idiosyncratic shocks to the depreciation of the capital stock. An important, and unfortunate, assumption is that inflation is iid, when in fact in the data, it is quite persistent. This can significantly distort inferences on how much inflation can inflate away the debt.[8] Therefore, the variance σ_π^2 must be thought of as significantly larger than just the measured variance in the data.

If $\sigma_\pi = 0$, then nothing of substance changes in the analysis. In fact, I had already allowed for this in the notation. Recall that r_t was pinned down as the solution to the model, independent of inflation, and then the price of the bonds v_t was determined by:

$$\frac{\dot{v}_t}{v_t} = \frac{r_t - \xi + \pi}{1 - \xi} \tag{26}$$

If inflation is higher, then bondholders who anticipate it, must be compensated with bond

[8] See Hilscher, Raviv and Reis (2014) for the interaction between the maturity of the debt and the persistence of inflation.

prices that rise faster over time. This is the only change in the model. But the government's ability to spend in real terms is unchanged.

Inflation uncertainty makes a difference. The appendix proves the following:

Lesson 8: *Expected inflation has no consequences on the spending ability of the government. An unexpected positive inflation shock allows the government to spend more from then onwards. But, higher variance of inflation lowers the persistent public spending that is feasible, and it lowers the upper bound on spending that the government can sustain.*

A large positive shock to inflation will lower the real value of the debt. Monetary policy will have deflated part of the debt, which loosens the budget constraint of the government, and so directly allows it to spend more in the future. At the same time, in the other direction, unexpectedly lower inflation raises the real value of the debt. On average these ex post effects cancel out.

Ex ante, however, a higher σ_π^2 makes the public debt less safe. Agents will wish to hold less of it. The safety component of the bubble premium gets smaller, and the interest rate on the bonds rises. The government now has to pay an inflation risk premium to bondholders that partly offsets the safety and store of value premium that it receives from them. The difference between g and r gets smaller, so the persistent deficits must therefore be smaller as well. At an extreme, if inflation is very volatile, then $g - r$ flips signs, and $S = 0$ so the public debt no longer can sustain a bubble.

Therefore, to loosen the debt burden on the fiscal authority, the best action for monetary policy in this economy would be to stabilize inflation as much as possible. This has a footprint on the government's budget, because it lowers the inflation risk premia that must be paid on the debt. Price stability generates fiscal resources through the bubble premium that can be spent on public services. This is especially important when $r < g$ because the government relies on the bubble premium to pay for persistent deficits. Avoiding that r rises above g is a crucial determinant of fiscal space, and monetary instability can trigger it and an accompanying fiscal crisis.

5.4 Financial repression: sacrificing private credit

A common form of financial repression is to force the financial system to hold underpriced government bonds. This is sometimes done by central banks that force banks to hold required reserves and do not pay interest on them. Other times, it is done by financial regulators that require financial institutions' assets to be held in safe investments for macro-prudential reasons, when in many countries the only safe asset is a liability from the government. In more extreme times, of war or after large expenses, government may legally or through strongly-stated moral suasion force financial markets to lend funds to support public programs at a fixed discounted rate.

By doing this, the government finances itself by imposing a repression premium on this forced debt. It interacts with the bubble premium. The appendix studies the trade-off between these two premia by changing the model in the following way. I now assume that public debt is split into voluntary and coerced debt: $b_t = b_t^v + b_t^c$. Voluntary debt must pay the equilibrium safe rate r_t, while coerced date instead pays a lower rate, which I set to zero. The government chooses coerced debt to grow at the growth rate of the economy g, so it does not asymptotically become negligible. The new policy choice is then the share of coerced debt in the total public debt: $b_t^c/(b_t^c + b_t^v)$

All households are forced to hold the same amount of coerced debt, but they are still free to choose how much voluntary debt to hold. Therefore, the choices of the households on the shares of consumption and savings in the different assets do not change relative to their voluntarily-disposed assets. The only change is that these assets are now $a_t - b_t^c$ every period, as if the private assets were lower. The appendix proves the following:

Lesson 9: *An increase in the share of coerced public debt raises the persistent public spending that is feasible and it may lower or raise the upper bound on persistent spending that the government can sustain.*

The direct impact of more repression is to cut the interest payments for the government. At the same time, this lower private investment in the economy by the entrepreneurs, which lowers the capital stock and the growth rate. On the side of financiers, their voluntary funds are first allocated to exhaust the borrowing constraint of the entrepreneurs, so it their voluntary holdings of public debt that fall. Since the interest rate rises, the bubble premium on the debt falls as well.

In the model, unambiguously, the fall in the bubble premium revenues does not offset the interest savings from the repression premium rising. At the same time, note that if tax revenues depended on economic activity, the decline caused by repression would automatically raise the deficit s. Moreover, repression affects the chances that there are financial crises, and these can require very large increases in spending s.[9]

[9] See Reis (2019) for the interaction of these three fiscal footprints of macroprudential policy.

6 Conclusion

Public debt is expected by 2021 to exceed 120% of GDP on average across the advanced economies, matching or exceeding the previous peak in the last 140 years, which had been hit in 1945. At the same time, interest rates relative to the growth rate of the economy are low in most advanced economies, even relative to a history where $r < g$ quite frequently.[10] This paper complemented the results in the enormous literature on public debt limits when the economy is dynamically efficient and there are no bubbles, with some lessons for the present day when $r < g$. Some of the results were surprising, while others less so, but all together they lay out clear policy trade-offs. Even if interest rates are quite low, this does not imply that there is a fiscal free lunch, and other policies may unexpectedly bring the public debt closer or further to their upper bound. To make the comparison starker with the previous literature, this paper ignored: the aggregate risk that, even if infrequently, r can jump and exceed g, making the deficit gamble fail; strategic sovereign default and how does $r - g$ affect the incentives to choose it; and whether private bubbles can also arise and how they compete with the public debt for private savings. Taking them all into account offers the intriguing possibility that economists may have to re-think the advice they give to countries to avoid a fiscal crisis.

[10] Sources: IMF Fiscal Monitor of October 2020, and Mauro and Zhou (2020).

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