# Wage Price Spirals

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December 2022

## 1 Introduction

What is a wage price spiral? In this paper, we use the expression "wage price spiral" to describe a mechanism, present also in standard new Keynesian models, that amplifies the effects of a given inflationary shock.

The basic logic of the mechanism is that workers and firms disagree on the relative price of goods and labor, that is on the real wage $W/P$. Due to this fundamental disagreement, when firms adjust nominal prices they try to reach a certain ratio $P/W$, when workers negotiate nominal wages they try to reach a different, higher ratio. The outcome is nominal inflation in both prices and wages. This interpretation of the wage price spiral highlights the presence of a distributional conflict as a proximate cause of inflation.

After defining the wage price spiral in this way, we ask some positive and normative questions. First, we ask whether the direction in which real wages move tells us something about the presence of a wage price mechanism. We argue that that is not the case. The total power of a wage price spiral, that is, its power to translate a given shock into higher (price and wage) inflation, is different from its relative power on the price and on the wage sides. Real wages can fall or increase in a wage price spiral, depending on the spiral's relative force on the two sides, not on its total force.

Second, we ask whether the direction in which real wages move tells us something about the nature of the shock hitting the economy. In particular, we ask whether a pure aggregate demand shock can cause real wages to decline. We show that this depends on properties of the economy when the shock hits. If the economy is in a state in which the supply of some inputs is relatively low and inelastic, and if there is limited substitution for those inputs, then a demand shock can trigger a price adjustment that is stronger than the wage adjustment, and cause a real wage decline. We call a demand shocks with these features a "supply constrained demand shock."

We show that the response of the economy to a supply constrained demand shock has similar qualitative features to its response to a supply shock in which the input supply is temporarily reduced and the central bank fails to adjust output to its lower natural level. In both cases, there is excess demand in the economy, which translates into a tension between the level of the real wage to which firms and workers aspire, and thus into a wage price spiral.

We then show that these two shocks display a similar pattern of adjustment in prices. The adjustment takes place in three phases. First, there is a bout of very high price inflation in the price of the inelastic non-labor inputs, followed by a gradual reduction in the price of these inputs. Second, there is a more persistent period of high good price inflation. Third, there is a smaller, but even more persistent increase in wage inflation. This pattern follows from our assumptions on the relative degree of price stickiness, with the input price being perfectly flexible and good prices being more flexible than wages. This pattern implies that at some point wage inflation crosses price inflation, so a period in which real wages fall is followed by a period in which they recover.

We then turn to normative questions and ask what is the optimal policy response to a supply shock coming from the scarce input. In particular, we ask two questions: could it be part of optimal policy to "run the economy hot", that is, to have a positive output gap and high inflation? Could it be part of the optimal policy to have a positive output gap and generalized inflation, that is, both high price and wage inflation?

The answer to the first question is positive. If the economy needs a lower real wage, it may be more efficient to reach the adjustment through high price inflation and a bit of wage deflation rather than though lower price inflation but deeper wage deflation. A positive output gap can help shift the adjustment in the direction of price inflation and in that way be socially beneficial.

The answer to the second question is more nuanced. On a point of theory, it is possible to construct examples in which, at some point, along the adjustment path, the output gap is positive and price and wage inflation are both positive. However, the argument for those examples strongly relies on sophisticated forward-looking behavior and on full commitment by the central bank. From a discretionary perspective, if both price and wage inflation are above target, reducing the output gap reduces all distortions in the economy, so a hot economy with both high price and wage inflation seems inconsistent with optimal policy.

### 1.1 Related literature

[TBD]

## 2 Model

The economy is a standard new Keynesian environment, set in continuous time, with Calvo assumptions on both price and wage setting as in Erceg et al. (2000). To capture supply shocks an important ingredient is the presence of a scarce input, with a flexible price, which enters the firms' CES production function with elasticity of substitution possibly different from one.[1] The classic example is to interpret this input as an energy primary product, but we also interpret it more broadly to capture shortages and bottlenecks in the supply of intermediates like microchips or lumber, which have appeared at different points during the pandemic recovery.

## 2.1 Setup

The representative household has preferences

$$\int_0^\infty e^{-\rho t}\left(\frac{1}{1-\sigma}C_t^{1-\sigma}-\frac{\Phi_t}{1+\eta}\int_0^1 N_{jt}^{1+\eta}dj\right)dt,$$

where $C_t$ is an aggregate of a continuum of varieties $C_t = \left(\int_0^1 C_{jt}^{1-1/\varepsilon}dj\right)^{\frac{1}{1-1/\varepsilon}}$, $N_{jt}$ is the supply of specialized labor of type $j$ and $\Phi_t$ is a labor supply shock. Each consumption variety $j$ is supplied by a monopolistic firm with the production function

$$Y_{jt} = F\left(L_{jt}, X_{jt}\right) \equiv \left(a_L L_{jt}^{\frac{\epsilon-1}{\epsilon}} + a_X X_{jt}^{\frac{\epsilon-1}{\epsilon}}\right)^{\frac{\epsilon}{\epsilon-1}},$$

where $L_{jt}$ is a labor aggregate made of all labor types $L_{jt} = \left(\int_0^1 L_{jkt}^{1-1/\zeta}dk\right)^{\frac{1}{1-1/\zeta}}$. Each labor type $k \in [0,1]$ is supplied by a monopolistic union that acts on behalf of the representative household. Integrating over firms the total employment of labor of type $k$ is $N_{kt} = \int_0^1 L_{jkt}dj$. The representative household owns an exogenous endowment $X_t$ of the input $X$ and sells it to the monopolistic good producers on a competitive market at the price $P_{Xt}$.

Let us focus on characterizing the price and wage setting conditions of firms and unions, assuming that each firm gets to reset its price following a Poisson process with rate $\lambda_p$ and each union gets to resets its wage following a Poisson process with rate $\lambda_w$. $P_t^*$ and $W_t^*$ denote the price set by the firms and unions that can update at time $t$ while $P_t$ and $W_t$ denote the price indexes for the good and labor aggregates.

Beginning from the firms, the nominal marginal cost of producting good $j$ can be expressed, from standard cost minimization, as

$$\frac{W_t}{F_L\left(L_{jt}, X_{jt}\right)} = \frac{W_t}{a_L Y_{jt}^{\frac{1}{\epsilon}} L_{jt}^{-\frac{1}{\epsilon}}},$$

[1]This is formally equivalent to having labor and capital, with capital renterd at a flexible price, although the interpretation is different.

or, in log-linear deviations from steady state

$$w_t - mpl_{jt} \tag{1}$$

where

$$mpl_{jt} = \frac{1}{\epsilon}\left(y_{jt} - l_{jt}\right)$$

is the marginal product of labor. The production function can be written in log-linear approximation

$$y_{jt} = s_L l_{jt} + s_X x_{jt}, \tag{2}$$

where $s_L$ is the labor share and $s_X$ is the share of input $X$ in steady state, with $s_L + s_X = 1$. All firms being price takers in the input market, they all hire labor and the input $X$ using the same ratio $L_{jt}/X_{jt}$ that satisfies the optimality condition

$$\frac{W_t}{P_{Xt}} = \frac{F_L\left(L_{jt}, X_{jt}\right)}{F_X\left(L_{jt}, X_{jt}\right)} = \frac{a_L}{a_X}\left(\frac{L_{jt}}{X_{jt}}\right)^{-\frac{1}{\epsilon}}.$$

Therefore, $l_{it} - x_{it} = l_t - x_t$ and the marginal product of labor is equalized across firms and satisfies

$$mpl_t = \frac{1}{\epsilon}\left(y_t - l_t\right) = \frac{s_X}{\epsilon}\left(x_t - l_t\right). \tag{3}$$

Also, using the optimality condition above, the price of the $X$ input can then be written as

$$p_{Xt} = w_t - \frac{1}{\epsilon}\left(x_t - l_t\right).$$

## 2.2 Price and wage setting

Optimal price setting implies that firms set their price at time $t$ as an average of future nominal marginal costs, conditional on not resetting. Since nominal marginal costs are given in (1), we get

$$p_t^* = \left(\rho + \lambda_p\right)\int_t^\infty e^{-\left(\rho+\lambda_p\right)(\tau - t)}\left(w_\tau - mpl_\tau\right)d\tau. \tag{4}$$

Similarly, we can derive the wage setting equation

$$w_t^* = \left(\rho + \lambda_w\right)\int_t^\infty e^{-(\rho+\lambda_w)(\tau - t)}\left(p_\tau + mrs_{\tau,t}\right)d\tau \tag{5}$$

where

$$mrs_{\tau,t} = \phi_\tau + \sigma y_\tau + \eta\left[n_\tau + \zeta\left(w_\tau - w_t^*\right)\right]$$

is the marginal rate of substitution between consumption and leisure at time $\tau$ for workers who reset their price at time $t \leq \tau$ (since their labor supply is $n_\tau + \zeta\left(w_\tau - w_t^*\right)$).

The two equations above is where a price-wage spiral appears in the model. Price setters aim to get a certain price to wage ratio in current and future periods, so they set their nominal price to catch up with current and anticipated future nominal wages. Symmetrically, wage setters aimt to get a certain wage to price ratio and so aim to catch up with current and future nominal prices.

The objectives of price setters and wage setters are captured, respectively, by $mpl$ and $mrs$. In a flexible price economy, the two equations above would boil down to $p_t - w_t = mpl_t$ and $w_t - p_t = mrs_t$, which then requires necessarily $mpl_t = mrs_t$. In a flexible price economy the aspirations of firms and workers for the relative price of goods and labor must necessarily be consistent with each other. In a sticky price economy, instead, these aspirations may be inconsistent and, depending on shocks and on policy responses, the economy can feature $mpl_t \neq mrs_t$. When that happens, workers and firms will try to set nominal prices to reach their desired relative price and, since these desired relative prices are inconsistent, the result will be inflation, as we shall see momentarily.

To go from equations (4) and (5) to wage and price inflation, let us combine them with the dynamic equations for the price and wage indices

$$\dot{p}_t = \lambda_p \left( p_t^* - p_t \right), \tag{6}$$

$$\dot{w}_t = \lambda_w \left( w_t^* - w_t \right). \tag{7}$$

As shown in the appendix, this leads to the following expressions for price and wage inflation

$$\rho \pi_t = \Lambda_p \left( \omega_t - mpl_t \right) + \dot{\pi}_t, \tag{8}$$

$$\rho \pi_t^w = \Lambda_w \left( mrs_t - \omega_t \right) + \dot{\pi}_t^w, \tag{9}$$

where

$$\omega_t = w_t - p_t$$

is the real wage, $\pi_t \equiv \dot{p}_t$ and $\pi_t^w \equiv \dot{w}$ are price and wage inflation, and

$$mrs_t = \phi_t + \sigma y_t + \eta n_t \tag{10}$$

is the cross-sectional average of the marginal rate of substitution between consumption and leisure. The coefficients $\Lambda_p$ and $\Lambda_w$ reflect how fast prices and wages adjust in response to deviations of the real wage from, respectively, $mpl_t$ and $mrs_t$, and their values are[2]

$$\Lambda_p \equiv \lambda_p \left( \rho + \lambda_p \right), \quad \Lambda_w = \lambda_w \frac{\rho + \lambda_w}{1 + \eta \zeta}.$$

Solving forward, these equations give price and wage inflation as functions of the path of

[2]The presence of the term $1/\left(1 + \eta \zeta\right)$ in $\Lambda_w$ reflects stragegic complementarity in wage setting. The model does not feature strategic complementarity in price setting, due to the assumption of constant returns to scale and a frictionless market for the $X$ input, but it is easy to allow for strategic complementarity by introducing an additional firm-specific fixed factor.

$mpl_t$ and $mrs_t$ and of the real wage $\omega_t$:

$$\pi_t = \Lambda_p \int_t^\infty e^{-\rho(s-t)} \left(\omega_s - mpl_s\right) ds, \tag{11}$$

$$\pi_t^w = \Lambda_w \int_t^\infty e^{-\rho(s-t)} \left(mrs_s - \omega_s\right) ds. \tag{12}$$

These equations show how the disagreement $mpl \neq mrs$ causes inflationary tensions. The first observation is that when $mpl \neq mrs$ there is no value of the real wage $\omega$ that can ensure that at the same time $\omega_t - mpl_t$ and $mrs_t - \omega_t$ are both zero. The second observation is that, due to price and wage stickiness the real wage $\omega_t$ will adjust gradually through the equation

$$\dot{\omega}_t = \pi_t^w - \pi_t. \tag{13}$$

So there is a feedback between the tensions in $\omega_t - mpl_t$ and $mrs_t - \omega_t$ and the value of $\omega_t$.

As an aside, notice that the simplest new Keynesian framework with flexible wages is a special case of our enviroment with $\lambda_w \to \infty$ and it also features a price wage spiral. With flexible wages the second equation becomes $\omega_t = mrs_t$, so price inflation is directly driven by the discrepancy $mrs_t - mpl_t$.

In the next section, we use the three conditions above to analyze the model predictions conditional on given paths of $mpl_t$ and $mrs_t$, solving for the real wage $\omega_t$. In the following sections, we go back to the full model and to the underlying shocks that determine $mpl_t$ and $mrs_t$ in general equilibrium.

# 3 Characterization of Real Wage Dynamics

We now characterize the equilibrium path of wages and prices for given paths of $mpl_t$ and $mrs_t$. This allows us to formalize the idea that the direction in which the real wage moves is a symptom of relative forces on the demand and supply side of the labor market.

Combining equations (8) to (13) gives a second order ODE for the real wage $\omega_t$

$$\ddot{\omega}_t = \rho\dot{\omega}_t + \left(\Lambda_p + \Lambda_w\right)\omega_t - \Lambda_p mpl_t - \Lambda_w mrs_t. \tag{14}$$

The next proposition solves the ODE and provides an analytical characterization of $\omega_t$.

**Proposition 1.** *The real wage satisfies the first order ODE*

$$\dot{\omega}_t = r_1\omega_t + \int_t^\infty e^{-r_2(s-t)} \left[\Lambda_p mpl_s + \Lambda_w mrs_s\right] ds, \tag{15}$$

*where $r_1$and $r_2$ are the roots of the quadratic equation*

$$r\left(r-\rho\right)=\Lambda_p+\Lambda_w,$$

*and satisfy $r_1<0<\rho<r_2$. Solving (15) gives the real wage as a function of $\{mpl_t\}_{t=0}^{\infty}$, $\{mrs_t\}_{t=0}^{\infty}$ and the initial condition $\omega_0$*

$$\omega_t=e^{r_1t}\omega_0+\int_0^{\infty}H_{t,s}\left(\Lambda_p mpl_s+\Lambda_w mrs_s\right)ds.$$

*where $H_{t,s}=\left(\min\left\{e^{r_1(t-s)},e^{r_2(t-s)}\right\}-e^{r_1t-r_2s}\right)/\left(r_2-r_1\right).$*

The second term in (15) shows that real wage dynamics are driven by anticipated pressures on the two sides of the labor market: the real wage increases when either labor demand is high (high $mpl$) or labor supply is low (high $mrs$).

The first term in (15) shows that the real wage tends to mean revert, due to $r_1<0$. The reason is that a high real wage increases firms' marginal cost $\omega-mpl$, increasing price inflation, and, at the same time, reduces workers' marginal cost of labor supply $mrs-\omega$, reducing wage inflation. Both forces reduce the real wage.

The labor market pressures and the mean reversion force captured in (15) shape the real wage response to different shocks, as we can see in some simple examples that can be analyzed using a phase diagram.

## 3.1 Phase diagrams

Suppose the economy is in steady state and at date 0 we have a permanent reduction of $mpl$ from zero to a constant value $\overline{mpl}<0$, while stays $mrs$ constant at zero.

The dynamics of $\omega$ are illustrated in the phase diagram of Figure 1. The stationary locus $\dot{\omega}=0$ coincides with the $x$ axis. The stationary locus $\ddot{\omega}=0$ is downward sloping, from (14). They are both drawn in purple. The saddle path in blue comes from 15, which in this example becomes

$$\dot{\omega}_t=r_1\omega_t+\frac{1}{r_2}\Lambda_p\overline{mpl}.$$

Setting $\dot{\omega}=0$ and using $-r_1r_2=\Lambda_p+\Lambda_w$, the new steady state for $\omega$ is

$$\overline{\omega}=\frac{\Lambda_p}{\Lambda_p+\Lambda_w}\overline{mpl}. \tag{16}$$

Therefore, in this example the wage falls from $\omega_0=0$, along the saddle path, asymptoting at $\overline{\omega}$.

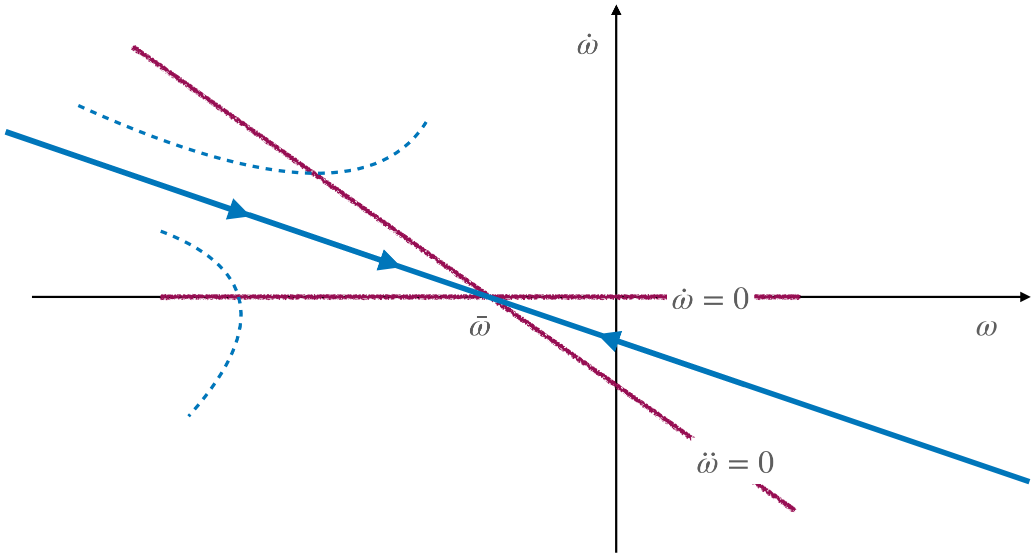


Figure 1: A permanent shock

What is driving down real wages along the path towards $\overline{\omega}$?

To understand the intuition it is useful to go back to price and wage inflation, recalling equations (11) and (12) above. Firms face higher marginal costs due to the lower $mpl$. The anticipation of lower real wages in future periods partly dampens this force, because marginal costs are $\omega - mpl$. However, the net effect remains positive because real wages, as shown in Figure 1, are always higher than $\overline{\omega}$, which, in turn, is higher than $\overline{mpl}$ from (16). Therefore, the expression $\omega - mpl$ is positive at all dates and decreasing over time. Inflation is then always positive, decreases over time, and converges to

$$\lim_{t\to\infty} \pi_t = \frac{\Lambda_p}{\rho}\left(\overline{\omega} - \overline{mpl}\right) = -\frac{1}{\rho}\frac{\Lambda_p \Lambda_w}{\Lambda_p + \Lambda_w}\overline{mpl}. \tag{17}$$

Wage inflation is also positive, because workers try to make up for lower real wages in the future by trying to bid up their nominal wages. However, it can be shown that wage inflation is lower than price inflation. Wage inflation increases over time and converges to $\Lambda_w \overline{\omega}/\rho$, which is equal to the long run inflation rate (17).

Due to the initial shift in $mpl$, firms and workers disagree on the real wage they would like to get in setting their nominal demands: workers aim for a real wage equal to $mrs = 0$, firms aim for $mpl < 0$. Price and wage inflation do not resolve the tension between workers' and firms' aims. But the decline in the real wage reallocates the tension from the firms' side to the workers' side, until, asymptotically, the tension is balanced, wage and price inflation are equal, and the real wage is $\overline{\omega}$. The level $\overline{\omega}$ is a weighted average of the objective of the two parties, with weights that depend on the speeds $\Lambda_p$ and $\Lambda_w$ at which price and wage inflation respond to deviations of $mpl$ and $mrs$ from $\omega$. To better visualize the weights, notice that following a permanent change in both $mpl$ and $mrs$, the

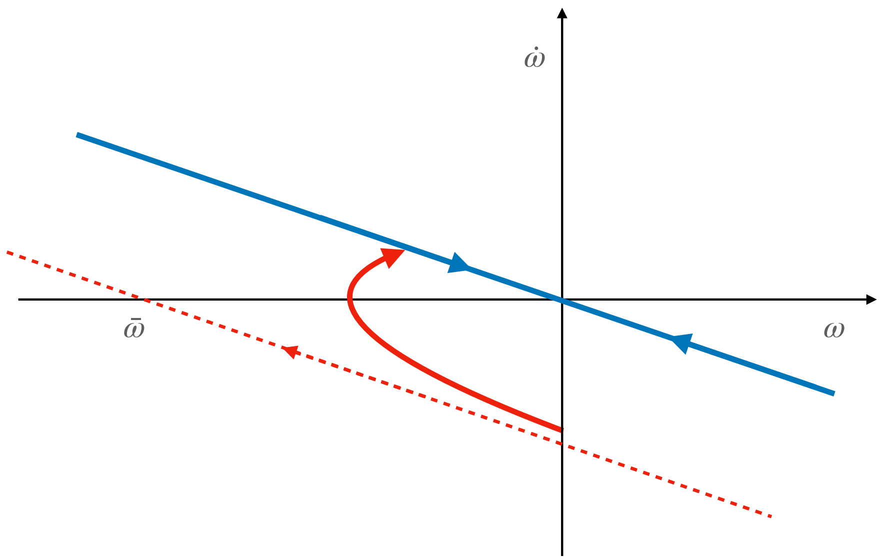


Figure 2: A transitory shock

expression for the long run real wage is

$$\overline{\omega} = \frac{\Lambda_p}{\Lambda_p + \Lambda_w}\overline{mpl} + \frac{\Lambda_w}{\Lambda_p + \Lambda_w}\overline{mrs}.$$

An example featuring a permanent gap between $mpl$ and $mrs$ is useful but extreme. If calibrated with a realistically low value of $\rho$, such an example yields very large levels of wage and price inflation for a given shock $\overline{mpl}$. This is just a reflection of the fact that the long-run new Keynesian Phillips curve is very steep. Therefore, let us turn to a temporary changes.

Consider an economy in steady state with all variables at 0. At $t = 0$, unexpectedly, firms realize that for a finite time interval $[0, T]$ they will face $\overline{mpl} < 0$. At $T$, $mpl$ goes back to zero. The value of $mrs$ remains at zero throughout.

The dynamics of $\omega$ following the shock are illustrated in Figure 2. First, the economy follows the red solid line, until that line meets the blue solid line at time $T$, then the economy follows the blue saddle path asymptoting back to the origin. The real wage first falls towards $\overline{\omega}$; at some point, before $T$, the real wage starts growing again, due to the increased strength of the mean-reverting force; finally, after the impulse to $mpl$ is gone, the real wage converges back to zero.

The intuition for these dynamics is closely related to the case of a permanent shift. Figure 3 shows the responses of $\pi, \pi_w$ and $\omega$ in a numerical example. In the interval $[0, T]$ real wages are below 0, but less so than productivity $mpl$. Therefore the marginal cost $\omega - mpl$ is positive in the interval $[0, T]$, driving up price inflation. Lower real wages are below the workers $mrs$, which is zero throughout, driving up wage inflation. The initial force

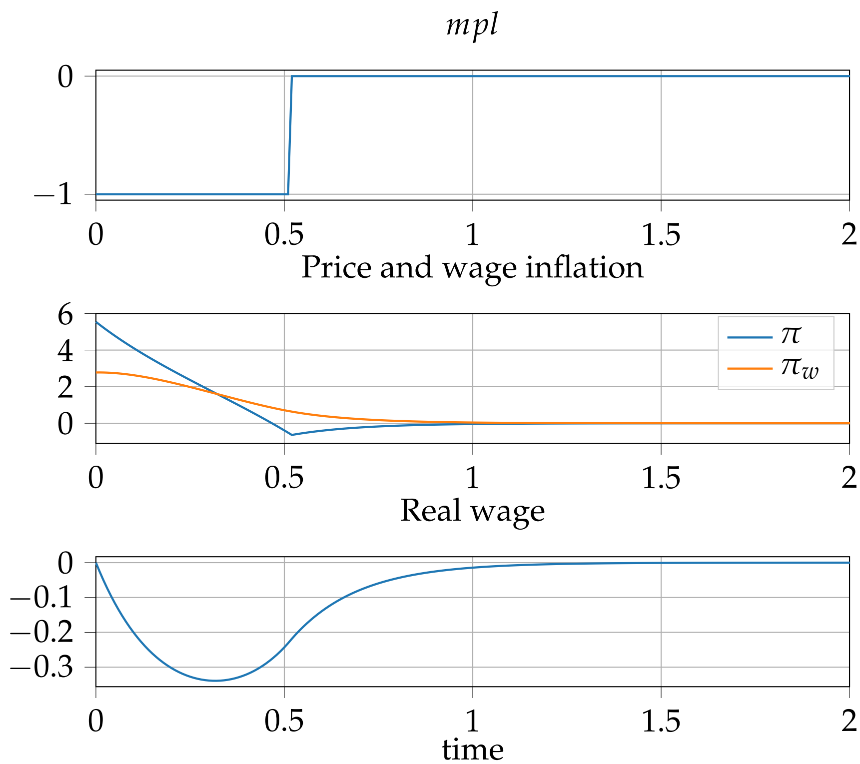


Figure 3: A temporary shift in $mpl$

on the price side is stronger, which is consistent with real wages falling. At some point, wage and price inflation cross and real wages start growing. This happens because as $T$ approaches firms anticipate lower marginal costs, due to $\omega - mpl < 0$ after $T$. At the same time the force on the wage side remains positive throughout.

Proposition 5 in the appendix provides formal derivations for a general class of experiments like the two just analyzed, in which only one side of the labor market is affected, that is, where only $mpl$ or only $mrs$ deviate from zero.

In most relevant cases, the underlying economic shocks change both $mpl$ and $mrs$ at the same time. In that case, the shape of the responses on the two sides may produce a variety of behaviors. In the next section, we focus on paths for $mpl$ and $mrs$ that decay exponentially over time.

# 4 Total and Relative Effects of Wage Price Spirals

Building on the results of the previous section, we can now turn to distinguishing the total effect of the wage price spiral mechanism on (wage and price) inflation, from its relative effect (wage vs price inflation).

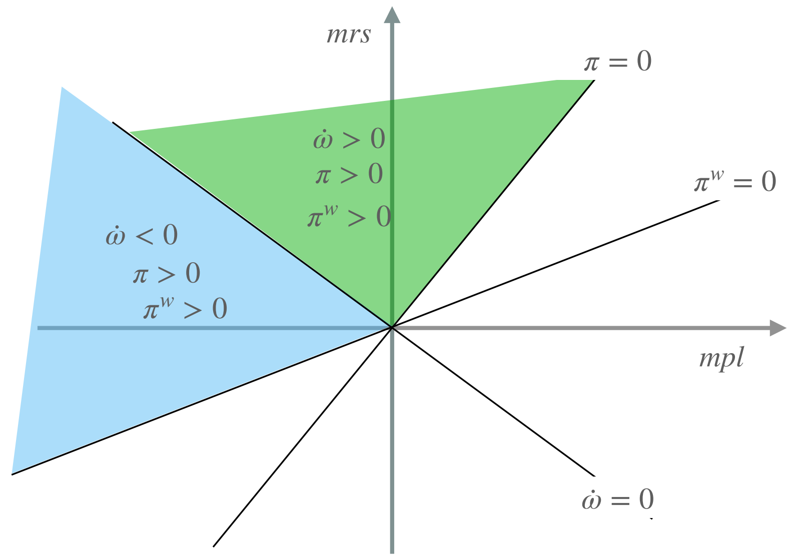


Figure 4: Regions for $mpl_0$ and $mrs_0$

Let's focus on shocks that produce exponentially decaying paths for $mpl$ and $mrs$. The economy is in steady state before time 0. At time 0, there is an unexpected shock and from then on the paths of $mpl$ and $mrs$ are

$$mpl_t = mpl_0 e^{-\delta t}, \quad mrs_t = mrs_0 e^{-\delta t},$$

where $\delta$ is the speed at which the shock dies out.

Define the coefficients

$$\psi = \frac{r_2}{r_2 + \delta} \frac{-r_1}{-r_1 + \rho}, \quad \kappa = \frac{\Lambda_p}{\Lambda_p + \Lambda_w},$$

which are both in $(0, 1)$.

**Proposition 2.** *Given exponentially decaying paths for $mpl$ and $mrs$, the effects on price and wage inflation at $t = 0$ are*

$$\dot{\omega}_0 > 0 \text{ iff } \Lambda_p \cdot mpl_0 + \Lambda_w \cdot mrs_0 > 0;$$
$$\pi_0 > 0 \text{ iff } mrs_0 > \frac{1 - \psi\kappa}{\psi(1-\kappa)} mpl_0;$$
$$\pi_0^w > 0 \text{ iff } mrs_0 > \frac{\psi\kappa}{1 - \psi(1-\kappa)} mpl_0;$$

*Where* $\frac{1-\psi\kappa}{\psi(1-\kappa)} > 1 > \frac{\psi\kappa}{1-\psi(1-\kappa)}$.

The proposition is illustrated in Figure 4, where we drop the time subscript 0 for readability. The green and blue regions are those in which the economy features both price and

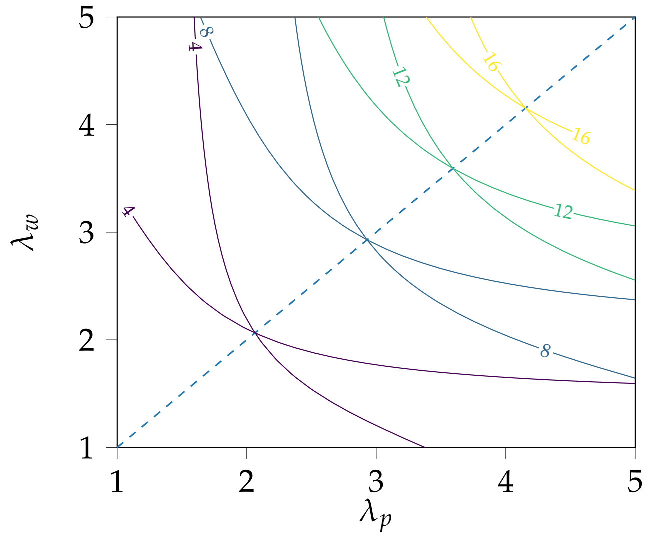


Figure 5: Price and wage inflation contours for different degrees of stickiness

wage inflation. Both $mrs_0 > 0$ and $mpl_0 < 0$ are inflationary forces, and we get inflation as long as one of them is present and strong enough. In particular, $mrs_0 > 0$ acts directly on workers' wage demands, $mpl_0 < 0$ acts directly on firms' price demands. Both also act indirectly. A higher $mrs_0$, by pushing up real wages tends to increase marginal costs and push up price inflation. A low $mpl_0$, by pushing real wages down, tends to increase wage demands and wage inflation. The fact that $mrs$ acts directly on wages, while $mpl$ acts directly on prices gives some intuition for why the slope of the $\pi = 0$ line is steeper than that of the $\pi^w = 0$ line. The difference between the green region and the blue region is that in the blue region the real wage declines at $t = 0$ while it increases in the green region. The reason for the difference is the relative strength of the pressure on price setters and wage setters.

Let us now do a different exercise: fix the size of the initial shocks $mrs_0 > 0$ and $mpl_0 < 0$ and change the economy's parameters to vary the degree by which the shocks get amplified. In particular, let us change the parameters $\lambda_p$ and $\lambda_w$. As we increase the speed at which wages and prices are reset, the wage price spiral mechanism gets stronger. This is shown in Figure 5, where we plot level curves for $\pi$ and $\pi_w$. The relatively steeper curves (in absolute value) correspond to $\pi$, the flatter ones to $\pi_w$. A higher frequency of price adjustment $\lambda_p$ increases both $\pi$ and $\pi_w$, but has a stronger effect on the former. The reverse holds for $\lambda_w$. For ease of illustration, we choose an economy with $\eta = 0$, hit by a symmetric shock $mrs_0 = -mpl_0$. This implies that $\lambda_p = \lambda_w$ implies $\Lambda_p = \Lambda_w$ and, from Proposition 2, it implies $\dot{\omega}_0 = 0$, which means that nominal price and wage inflation are equal, $\pi_0 = \pi_{w0}$. This is confirmed in the figure, where the contour levels corresponding to equal price and wage inflation meet on the 45 degree line.

Increasing either price or wage flexibility increases *both* price and wage inflation. This is what we call the total force of the wage price mechanism. At the same time, what happens to the real wage depends on the relative force on the two sides. Increasing $\lambda_p$ tends to move us to the region below the 45 degree line, where real wages fall. Increasing $\lambda_w$ has the opposite effect. This is what we mean by the relative power of the mechanism.

# 5 Demand and Supply Shocks

We now go back to the full model and trace back price and wage inflation to the general equilibrium effect of underlying shocks.

We focus on two shocks. First an expansionary demand shock, driven by easy monetary policy (easy fiscal policy would have similar implictions in our setup).

A common view is that excessive demand would work its way from a tight labor market, to higher wages, to higher prices. Following this intuition a pure demand shock should manifest itself in increasing real wages. We show that in our model general equilibrium forces are at work on both sides of the labor market and that the direction of adjustment of the real wage is in general ambiguous. This is especially true when the scarce, inelastic input $X$ plays an important role.

Consider a monetary shock that leads to a temporary increase in employment $n_0 > 0$ on impact, the shock decays exponentially at rate $\delta$, so

$$n_t > n_0 e^{-\delta t}.$$

The responses of $mpl_t$ and $mrs_t$ are easily derived from (3) and (10):

$$mpl_t = -\frac{s_X}{\epsilon} e^{-\delta t} n_0, \quad mrs_t = (\sigma s_L + \eta)\, e^{-\delta t} n_0.$$

Substituting in the conditions of Proposition 2 shows that price and wage inflation are both positive following the shock. What happens to the real wage, though, is in general ambiguous. The following is an immediate corollary of Proposition 2.

**Proposition 3.** *In response to a monetary shock that leads to a transitory increase in employment, real wages fall on impact if and only if*

$$\Lambda_p \frac{s_X}{\epsilon} > \Lambda_w (\sigma s_L + \eta)\, .$$

The left-hand side of the inequality captures direct effects on price inflation. This term depends on the effect of higher employment on marginal costs and on stickiness in price setting, captured by $\Lambda_p$. The effect of employment on marginal costs is larger when the scarce input $X$ is more important in the production of the final good (higher share $s_X$) and when the elasticity of substitution between labor and $X$ is lower. The term on the right-hand side captures direct effects on wage inflation. This term depends on the effect on the marginal rate of substitution and on stickiness in wage setting, captured by $\Lambda_w$. The

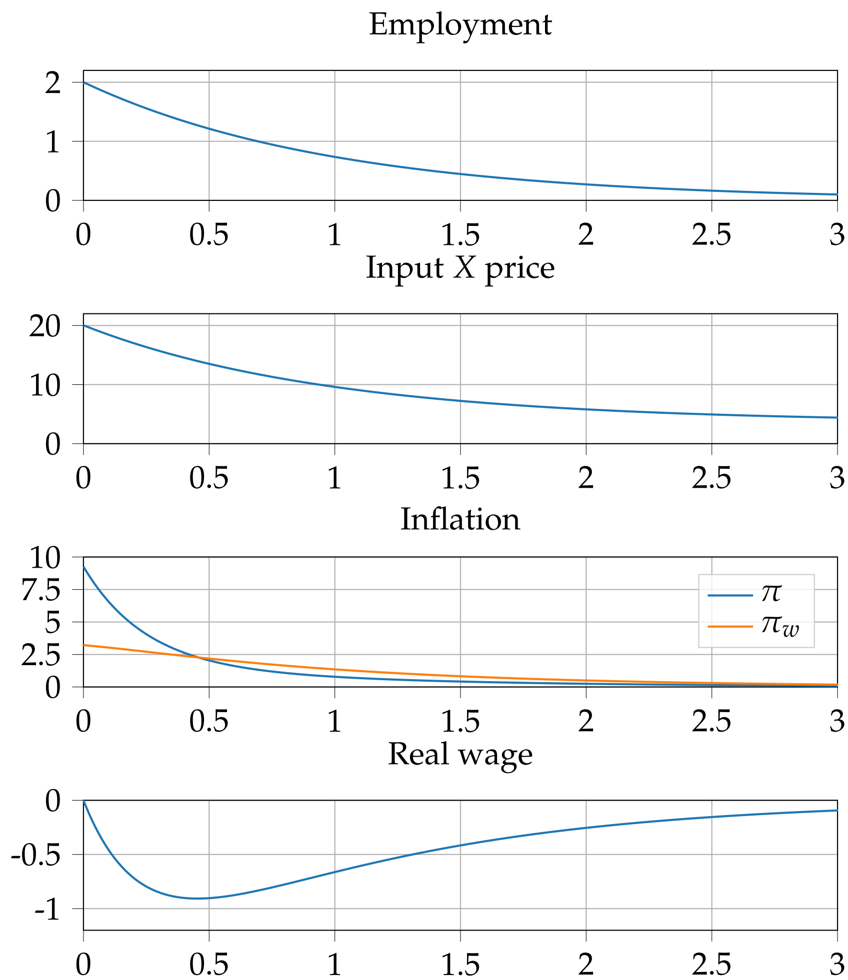


Figure 6: A supply-constrained demand shock

effect on the marginal rate of substitution, in turn, depends on an income effect, captured by the term $\sigma s_L$, since $s_L$ is the elasticity of output to the labor input, and on the inverse Frisch elasticity $\eta$.

Overall, if the effect on firms' marginal costs is relatively stronger than the effects on workers' marginal rate of subsitution and if prices are relatively more flexible than wages, we get a reduction in real wages.

In Figure 6 we plot the response to a temporary expansionary shock that increases $n$ above its potential level by 2%, with a decay $\delta = 1$ in a simple numerical example.[3] The parameters used are in the Table 1.

The first panel shows the shock to $n$. The remaining panels show the responses of different prices.

[3]All plots show log deviations from steady state times 100, or, approximately, percentage deviations from steady state.

| | | | |
|---|---|---|---|
| Preferences | $\sigma = 1$ | $\eta = 0$ | $\rho = 0.05$ |
| Technology | $s_X = 0.1$ | $\epsilon = 0.1$, | |
| Stickiness | $\lambda_p = 4$ | $\lambda_w = 1$ | |

Table 1: Parameters

The input price is flexible, so it jumps on impact and then gradually goes back to its initial level, as the shock goes away. This is shown in the second panel of the figure. Notice that this panel shows the level of the input price, not its rate of inflation. Due to perfect flexibility $P_X$ jumps by 20% at $t = 0$. This large increase is due to our assumption of a low elasticity of substitution between labor and the input $X$ ($\epsilon = 0.1$), so when the employment is growing too fast relative to the supply of $X$, the price of $X$ reacts strongly.

The effect of the increase in the input price is to increase firm's marginal costs. The impact effect on the nominal marginal cost $w_0 - mpl_0$ is 2%, as the input represents 10% of the cost in steady state ($s_X = 0.1$). This impulse translates into fast inflation on impact, due to our assumption of relatively flexible prices ($\lambda_p = 4$, i.e, prices reset every quarter). This is plotted in the third panel.

Wages respond because high employment translates into high real wage demands. In our simple model with $\eta = 0$, this is only due only to an income effect: as consumption grows, workers need higher wages to be induced to work. For illustration we have chosen parameters such that the impact effect on the nominal marginal cost of labor $p_0 + mrs_0$ is identical to the effect on the marginal cost of goods, both are 2%. However, wages are more sticky ($\lambda_w = 1$), so the effect on wage inflation is weaker. Wage inflation is also plotted in the third panel. The conditions for Proposition 3 are satisfied and the real wage falls on impact, as shown in the fourth panel.

To be clear, this is just a numerical example with numbers chosen for clarity of illustration. Nonetheless, there is clear qualitative feature that we want to highlight: the adjustment happens in three phases.

1. First, there is a bout of very fast inflation in the sector where the supply constraints are binding, here the market for input $X$.
2. Second, there is a phase in which price inflation is faster than wage inflation, as price setters react relatively quickly to the increase in input costs.
3. At some point (near $t = 0.5$ in our example) wage inflation crosses price inflation and we enter the third phase in which real wages recover. The input scarcity is going away, so the pressure on firms' marginal costs is weaker, while workers are still trying to catch up to the higher cost of living, given their real wage aspirations.

Consider now the same economy's response to a supply shock due to a temporary reduction in $x$. Suppose for now that the central bank responds in such a way as to keep

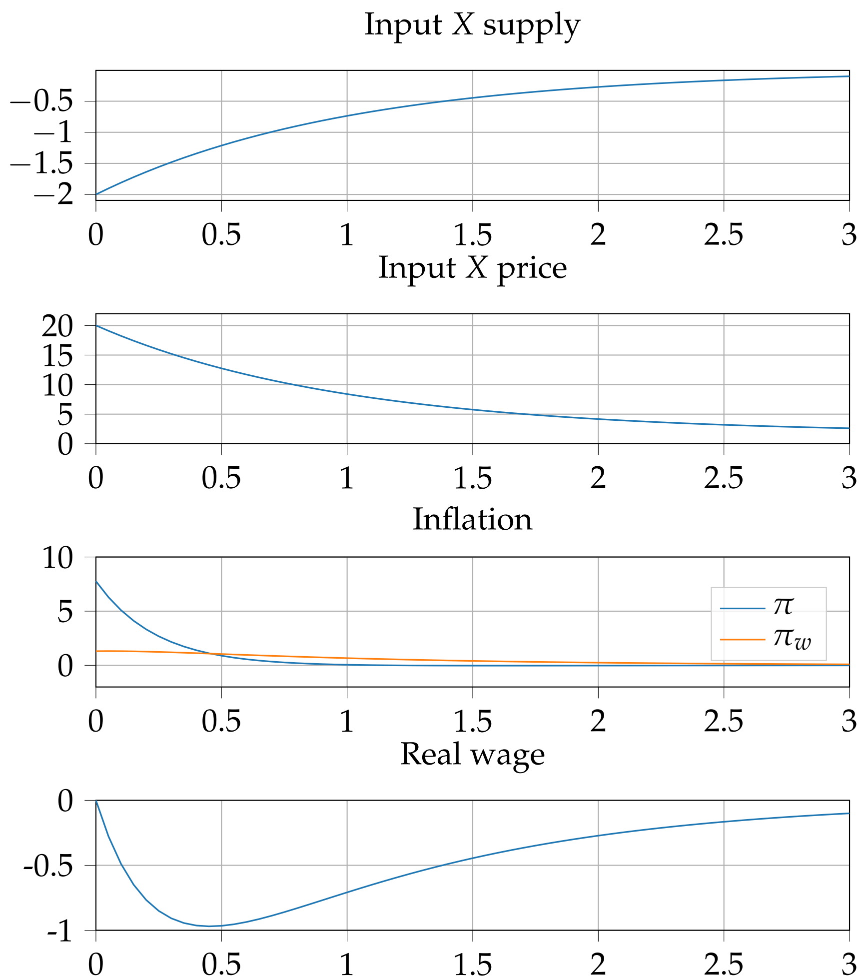


Figure 7: A supply shock

employment constant at $n_t = 0$. The responses of $mpl$ and $mrs$ are now

$$mpl_t = \frac{s_X}{\epsilon} e^{-\delta t} x_0 < 0, \quad mrs_t = \sigma s_X e^{-\delta t} x_0 < 0.$$

The main difference is that now the reduction in output reduces workers' $mrs$, via an income effect. This weakens real wage demands. However, given our parameter choices, the inflationary forces on the firms' side are still strong enough that we obtain positive wage and price inflation. In the representation of Figure 4 we are in the portion of the blue region that intersects with the lower left quadrant. From Proposition 2, we also know that $mpl_0 < 0$ and $mrs_0 < 0$ implies that the real wage falls on impact for any parameter configuration.

The responses are illustrated in Figure 7. For ease of comparison, we pick a negative shock to $x_0$ that produces the same increase in the input price as the positive $n_0$ shock in the demand shock exercise of Figure 6.

While nominal wages are growing less and the real wage drop is larger than in Figure 6, there is a common element to the demand and supply shocks just analyzed: the three-phase adjustment discussed above is qualitatively the same.

The response to the supply shock depend on how monetary policy adjusts. So far, we assumed a policy that keeps the employment path unchanged. However, the natural level of employment depends in general on $x_t$. In particular, keeping employment and output at their the natural level requires $mrs_t = mpl_t$, and so $n_t^*$ can be derived from the condition

$$\sigma\left(s_N n_t^* + s_X x_t\right) + \eta n_t^* = \frac{s_X}{\epsilon}\left(x_t - n_t^*\right).$$

The responses of price and wage inflation when

$$n_t = n_t^* = \frac{\frac{1}{\epsilon} - \sigma}{\sigma\left(s_N + \frac{s_X}{\epsilon}\right) + \eta} s_X x_t$$

are plotted in Figure 8. Since our parametrization features a low degree of substitutability between labor and the input $X$, we have $\frac{1}{\epsilon} - \sigma > 0$ and a reduction in $x_t$ lowers the natural level of employment, as shown in the first panel. The natural level of output $y_t^* = s_X x_t + s_N n_t^*$ is then lower for two reason, the direct effect of a lower $x_t$ and for the lower level of natural employment. There is a clear difference in the inflation paths when quantities are at their natural levels: we see positive price inflation, but negative wage inflation. This goes on as long as the real wage falls, once the real wage starts growing again, the signs of price and wage inflation flip. In other words, real wage adjustments always take place with nominal prices and wages moving in opposite directions.

This is not just an outcome of our choice of parameters. When quantities are at their natural level we have $mrs_t = mpl_t = \omega_t^*$ and the inflation equations become

$$\pi_t = \Lambda_p \int_t^\infty e^{-\rho(s-t)}\left(\omega_s - \omega_s^*\right) ds,$$
$$\pi_t^w = \Lambda_w \int_t^\infty e^{-\rho(s-t)}\left(\omega_s^* - \omega_s\right) ds.$$

The following general result follows immediately.

**Proposition 4.** *If quantities are at their natural level, price and wage inflation $\pi_t$ and $\pi_t^w$ are either zero or have opposite sign.*

This result can be visualized in the diagram of Figure 4, by noticing that the regions where $\pi$ and $\pi^w$ have the same sign are either entirely above or entirely below the 45 degree line, where $mrs = mpl$.

Comparing Figures 7 and 8 also shows that while employment falls more at the natural allocation, real wages fall less. This may seem surprising, but it is due to the fact the dynamics of the real wage are more strongly affected by $mpl$ than by $mrs$, and $mpl$ is higher along the path with lower employment. A different intuition for the same phenomenon

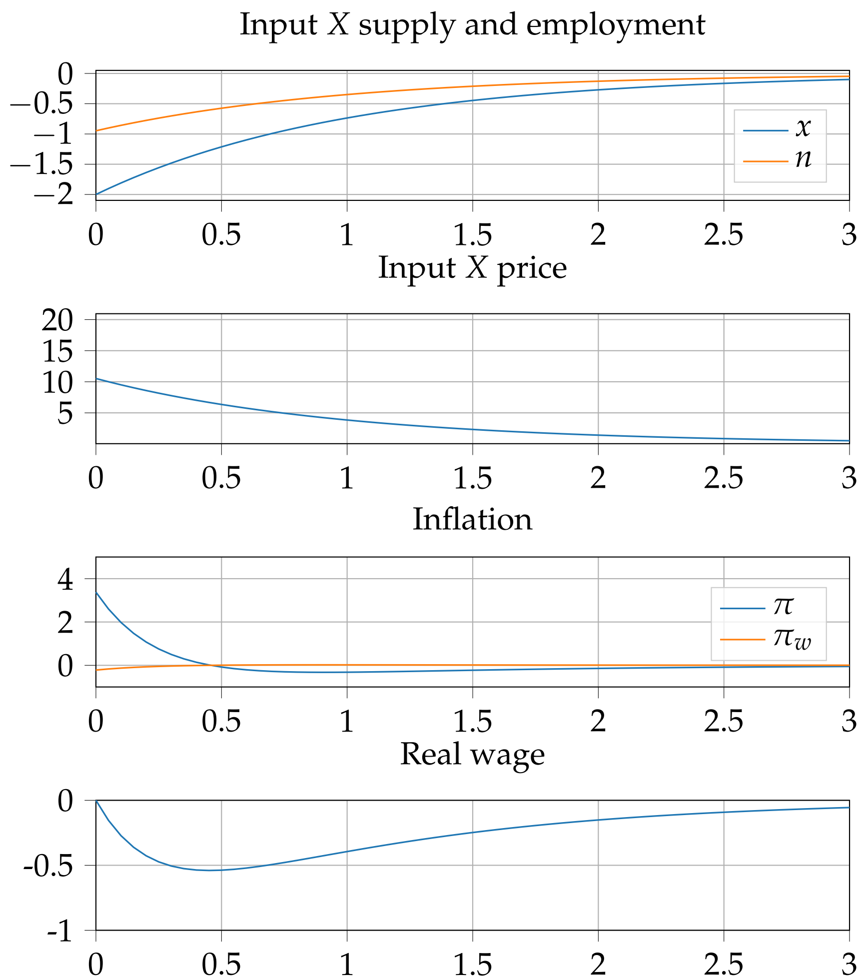


Figure 8: A supply shock with quantities on their natural path

is that lower employment reduces the pressure on the market for the scarce input, as seen in the second panel, weakening good inflation due to the high $X$ price and increasing the real wage.

To summarize the findings of this section, there is a common adjustment pattern, illustrated in Figures 6, 7, that may be caused either by a positive demand shock or by an insufficient demand contraction in response to a negative supply shock. This adjustment pattern shows both price and wage inflation, with price inflation stronger early on and wage inflation catching up later. If the central bank keeps always the economy at its flexible price allocation this pattern is not present, as price and wage inflation have opposite signs.

However, as it's well known, an economy with both price and wage rigidities does not feature "divine coincidence," so a policy of keeping quantities at their flexible price levels is not necessarily optimal in our environment. In the next section, we turn to optimal policy.

# 6 Optimal Policy

In the previous section, we looked at economies in which the central bank unnecessarily stimulates the economy (demand shock) or in which the central bank responds weakly to a supply shock, so as to allow for both price and wage inflation (the supply shock with $n_t = 0$). The first example is a policy mistake, by construction. Of course, due to imperfect information and lags in monetary policy, similar mistakes can happen. However, in this section we focus on the second scenario, a supply shock, and ask what is the optimal response in that scenario, even if monetary policy has perfect information on the underlying shocks and direct control on total spending.

The questions we address in this section here are two: is it possible that following a supply shock the optimal response is to let the economy overheat, that is, to choose a positive output gap $y_t - y_t^* > 0$? Is it possible that the optimal response entails both positive price and wage inflation?

It is well known that divine coincidence fails in our environment. But that is only a statement about feasibility, $\pi_t = \pi_t^w = 0$ and $y_t = y_t^*$ are simply not feasible in our economy because the real wage needs to move in the flexible price equilibrium and that is incompatible with zero nominal inflation in prices and wages. Our contribution here is to focus on a supply shock in our input-constrained economy and to aim to characterize the sign of possible deviations from divine coincidence. In particular, Proposition 5 in the previous section tells us that if the central bank chooses $y_t = y_t^*$, the necessary relative price adjustments in $w_t - p_t$ are never achieved by having *both* price and wage inflation. However, that proposition does not tell us what is the optimal response. That is the question we address here.

## 6.1 Optimal policy problem

Following standard steps, the objective function of the central bank can be derived as a quadratic approximation to the social welfare function and is

$$\int_0^\infty e^{-\rho t} \frac{1}{2} \left[ (y_t - y_t^*)^2 + \Phi_p \pi_t^2 + \Phi_w \left(\pi_t^w\right)^2 \right] dt. \tag{18}$$

The value of the coefficients $\Phi_p$ and $\Phi_w$ depend on the model parameters and are derived and reported in the appendix.

Defining the natural real wage

$$\omega_t^* = \frac{s}{\epsilon} \frac{\sigma + \eta + (\sigma - 1)\frac{s}{\epsilon}}{\sigma\left(1 - s + \frac{s}{\epsilon}\right) + \eta} x_t$$

we can express $mpl$ and $mrs$ as follows

$$mpl_t = \omega_t^* - \frac{s}{\epsilon}\left(n_t - n_t^*\right),$$

$$mrs_t = \omega_t^* + \left(\sigma\left(1-s\right) + \eta\right)\left(n_t - n_t^*\right).$$

The optimal policy problem is then to maximize (18), subject to the following constraints

$$\rho\pi_t = \Lambda_p\left(\omega_t - mpl_t\right) + \dot{\pi}_t,$$

$$\rho\pi_t^w = \Lambda_w\left(mrs_t - \omega_t\right) + \dot{\pi}_t^w,$$

$$\dot{\omega}_t = \pi_t^w - \pi_t.$$

### 6.2 Examples

We now consider a number of examples that illustrates the possible outcomes, following a supply shock, at the optimal policy.

[TO BE COMPLETED]

# Appendix

## Derivation of equations (8) and (9)

To derive (8), take time derivatives on both sides of (4) to get

$$\dot{p}_t^* = -\left(\rho + \lambda_p\right)\left(w_t - mpl_t\right) + \left(\rho + \lambda_p\right)p_t^*.$$

Next, take time derivatives on both sides of (6). Substituting the expression just derived for $\dot{p}_t^*$ and adding and subtracting $\left(\rho + \lambda_p\right)p_t$ on the right-hand side yields

$$\begin{aligned}\dot{\pi}_t &= \lambda_p\left(-\left(\rho + \lambda_p\right)\left(w_t - p_t + mc_t\right) + \left(\rho + \lambda_p\right)\left(p_t^* - p_t\right) - \pi_t\right) = \\ &= -\lambda_p\left(\rho + \lambda_p\right)\left(w_t - p_t + mc_t\right) + \rho\pi_t,\end{aligned}$$

where the second equality uses $\lambda_p\left(p_t^* - p_t\right) = \pi_t$. Rearranging gives (8).

To derive (9), first rewrite (5), substituting for $w_t^*$, as

$$w_t^* = \frac{\rho + \lambda_w}{1 + \eta\zeta}\int_t^\infty e^{-(\rho+\lambda_w)(\tau-t)}\left(p_\tau + mrs_\tau + \eta\zeta w_\tau\right)d\tau.$$

Taking time derivatives on both sides gives

$$\dot{w}_t^* = -\frac{\rho + \lambda_w}{1 + \eta\zeta}\left(p_t + mrs_t + \eta\zeta w_t\right) + \left(\rho + \lambda_w\right) w_t^*.$$

Taking time derivatives on both sides of (7) and substituting for $\dot{w}_t^*$ yields

$$\dot{\pi}_t^w = \lambda_w \left( -\frac{\rho + \lambda_w}{1 + \eta\zeta}\left(p_t + mrs_t + \eta\zeta w_t\right) + \left(\rho + \lambda_w\right) w_t + \left(\rho + \lambda_w\right)\left(w_t^* - w_t\right) - \pi_t^w \right) =$$
$$= -\lambda_w \frac{\rho + \lambda_w}{1 + \eta\zeta}\left(p_t - w_t + mrs_t\right) + \rho\pi_t^w,$$

which corresponds to (9).

## Proof of Proposition 1

Consider the second order non-autonomous ODE

$$\ddot{\omega}_t - \rho\dot{\omega}_t - \Lambda\omega_t = f_t,$$

which is the general version of (14) with

$$\Lambda = \Lambda_p + \Lambda_w, \quad f_t = -\Lambda_p mpl_t - \Lambda_w mrs_t$$

With $\Lambda > 0$ there are two real eigenvalues $r_1, r_2$ that solve

$$r^2 - \rho r - \Lambda = 0,$$

or, equivalently, that satisfy $r_1 + r_2 = \rho$ and $r_1 r_2 = -\Lambda$. Then the ODE can be written as

$$\left(\partial - r_2\right)\left(\partial - r_1\right)\omega_t = f_t$$

where $\partial$ is the time-derivative operator. Notice that

$$\left(\partial - r_2\right)\int_t^\infty e^{-r_2(s-t)} f_s ds = -f_t$$

so we get

$$\left(\partial - r_1\right)\omega_t = \frac{1}{\partial - r_2} f_t = -\int_t^\infty e^{-r_2(s-t)} f_s ds,$$

which gives (15). Integrating backwards gives

$$\omega_t = e^{r_1 t}\omega_0 + \int_0^t e^{r_1(t-s)} \int_s^\infty e^{-r_2(\tau - s)} \left[\Lambda_p mpl_\tau + \Lambda_w mrs_\tau\right] d\tau ds,$$

and changing the order of integration gives

$$\begin{aligned}
\omega_t &= e^{r_1 t}\omega_0 + e^{r_1 t}\int_0^t e^{-r_2\tau}\left[\Lambda_p mpl_\tau + \Lambda_w mrs_\tau\right]\int_0^\tau e^{(r_2-r_1)s} ds d\tau + \\
&+ e^{r_1 t}\int_t^\infty e^{-r_2\tau}\left[\Lambda_p mpl_\tau + \Lambda_w mrs_\tau\right]\int_0^t e^{(r_2-r_1)s} ds d\tau = \\
&= e^{r_1 t}\omega_0 + \int_0^t \frac{e^{r_1(t-s)} - e^{r_1 t - r_2 s}}{r_2 - r_1}\left(\Lambda_p mpl_s + \Lambda_w mrs_s\right) ds + \\
&+ \int_t^\infty \frac{e^{r_2(t-s)} - e^{r_1 t - r_2 s}}{r_2 - r_1}\left(\Lambda_p mpl_s + \Lambda_w mrs_s\right) ds.
\end{aligned}$$

## General Result for One-side Changes in *mrs* and *mpl* 2

The following result focuses on the effects of shocks that exclusively affect the labor demand side or the labor supply sidfe of the model, in the sense that they perturb $mpl_t$ without affecting $mrs_t$, or, vice versa.

**Proposition 5.** *Suppose there is no change in* $mrs_t = 0$ *and the path for* $mpl_t$ *is negative for all* $t \in [0, \infty)$. *Then the impact responses at* $t = 0$ *are*

$$\pi_0 > \pi_0^w > 0.$$

*Suppose there is no change in* $mpl_t = 0$ *and the path for* $mrs_t$ *is positive for all* $t \in [0, \infty)$. *Then the impact responses at* $t = 0$ *are*

$$\pi_0^w > \pi_0 > 0.$$

## Proof of Proposition 2

We first derive the real wage path using the expression in Proposition 1. Solving the integrals gives

$$\int_0^t e^{r_1(t-s)}\int_j^\infty e^{-r_2 j} e^{-\delta(s+j)} dj ds = \frac{1}{r_2+\delta}\int_0^t e^{r_1(t-s)-\delta s} ds = \frac{1}{r_2+\delta}\frac{e^{r_1 t} - e^{-\delta t}}{r_1+\delta},$$

and then

$$\omega_t = \frac{1}{r_2+\delta}\frac{e^{r_1 t} - e^{-\delta t}}{r_1+\delta}\left(\Lambda_p mpl_0 + \Lambda_w mrs_0\right).$$

To derive price inflation we solve the expression $\int_t^\infty e^{-\rho(\tau-t)}\omega_\tau d\tau$ in (11), using the following

$$\int_t^\infty e^{-\rho(\tau-t)}\frac{1}{r_2+\delta}\frac{e^{r_1\tau}-e^{-\delta\tau}}{r_1+\delta}d\tau =$$
$$\int_0^\infty e^{-\rho j}\frac{1}{r_2+\delta}\frac{e^{r_1(t+j)}-e^{-\delta(t+j)}}{r_1+\delta}dj =$$
$$\frac{1}{r_1+\delta}\frac{1}{r_2+\delta}\left(\frac{e^{r_1 t}}{-r_1+\rho}-\frac{e^{-\delta t}}{\delta+\rho}\right).$$

We then get that $\pi_t > 0$ if and only if

$$\frac{1}{r_1+\delta}\frac{1}{r_2+\delta}\left(\frac{e^{r_1 t}}{-r_1+\rho}-\frac{e^{-\delta t}}{\delta+\rho}\right)\left[\Lambda_p mpl_0+\Lambda_w mrs_0\right] > \frac{e^{-\delta t}}{\rho+\delta}mpl_0,$$

which can be rewritten using $-r_1 r_2 = \Lambda_p + \Lambda_w$ (from the proof of Proposition (1)), to get

$$\frac{r_2}{r_2+\delta}\frac{-r_1}{r_1+\delta}\left(\frac{e^{r_1 t}}{-r_1+\rho}-\frac{e^{-\delta t}}{\delta+\rho}\right)\frac{\Lambda_p mpl_0+\Lambda_w mrs_0}{\Lambda_p+\Lambda_w} > \frac{e^{-\delta t}}{\rho+\delta}mpl_0.$$

Setting $t = 0$ and rearranging gives the condition for $\pi_0 > 0$ in the statement of the proposition. Similar steps starting from equation (12), give the condition for $\pi_0^w > 0$.

# References

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